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Folk Theorem

Let G be given. Let $V = F(G) \cap SIR(G)$. If G satisfies a regularity condition (sufficient: $F(G)$ is I dimensional), then for each $v \in V$ and $\varepsilon > 0$ there is a $\delta_0 > 0$ such that the infinitely repeated version of G with discount factor $\delta > \delta_0$ has a subgame perfect equilibrium payoff v^* such that $|v - v^*| < \varepsilon$.
Informally: in an infinitely repeated game between patient players any individually rational and feasible payoff can be a subgame perfect equilibrium payoff.

Sketch of Proof

Need to think about three types of payoff:

1. The target payoff v^*.
2. The minmax payoffs $\underline{v}^i$ (one payoff vector associated with each player).
3. Conditional reward payoffs, $v'(i)$ defined to be

$$v'(i) = (\tilde{v}_1 + \nu, \ldots, \tilde{v}_{i-1} + \nu, \tilde{v}_i, \tilde{v}_{i+1} + \nu, \ldots, \tilde{v}_I + \nu)$$

   where $\tilde{v}$ such that $\tilde{v}_i \in (\underline{v}^i_i, v^*_i)$, $\nu > 0$, $v'(i)$ feasible for all i.

- Phase I: Equilibrium Path: play to get target payoffs v^*.
- Phase II: Punishment Path: If there is a deviation by more than one player, ignore it. If there is a deviation by Player i, minmax i for N periods.
- If anyone deviates during punishment, punish the deviator.
- Phase III: If not, play to get payoffs $v'(i)$.

Technicality

If there is a "public randomization device," then players can correlate strategies in a way that generates v^* and $v'(i)$ and $\underline{v}^i$. Otherwise, they can cycle through pure strategies in a way that approximates v^*. Doing so makes the proof messier and a bit trickier, but is not interesting. I'll ignore the problem.

Phase I

The strategies generate the desired payoff (essentially by definition). The problem is to confirm that there are no incentives to deviate.

In Phase I playing according to equilibrium yields v_i^*. A single deviation yields (potentially) a one shot payoff of M, the largest payoff in G, followed by N periods of $\underline{v}_i^i$, followed by v_i forever. Hence we need:

$$v_i^* \geq (1-\delta)M + \delta(1-\delta^N)\underline{v}_i^i + \delta^{N+1}v_i'.$$

As long as $v_i^* \geq v_i'$, this inequality will hold for N large and δ close to one.

Phase II

Must check that neither i nor $j \neq i$ want to deviate. For i the comparison is (when k periods of punishment remain):

$$(1-\delta^{k+1})\underline{v}_i^i + \delta^{k+1} v_i'(i) \geq (1-\delta)\underline{v}_i^i + \delta(1-\delta^N)\underline{v}_i^i + \delta^{N+1} v_i'(i)$$

For j the comparison is:

$$(1-\delta^{k+1})\underline{v}_j^i + \delta^{k+1} v_j'(i) \geq (1-\delta)M + \delta(1-\delta^N)\underline{v}_j^j + \delta^{N+1} v_j'(j)$$

The first inequality guarantees that a deviator does not gain from deviating from her punishment. The intuition is that the punishment phase is finite. A deviator would prefer to end the punishment quickly rather than take an immediate reward and re-start the punishment.

The second inequality guarantees that a punisher is willing to participate in the punishment. This is the critical point at which subgame perfection comes into play. The potential problem is that in order to punish i, another player must settle for a payoff that is lower than his individually rational payoff. This could only happen if latter he is rewarded for punishing. That is why we need a phase three payoff that is relatively high for the punishers.

Phase II Algebra

The verification is straightforward. In the case of a deviation by i, there is a potential one-period gain, but at the expense of (a) longer punishment and (b) postponing the final period.
In the case of a deviation by j, there is a potential one period gain and a potential gain because punishing may lead to a payoff worse than j's minmax. These gains are dominated for patient players because the final phase gives a uniformly higher payoff to j if she doesn't deviate.

Phase III

The comparison from Phase I establishes that deviations are not attractive here.

Implicit Assumption

I assumed that if there exists a profitable deviation, then there exists a profitable deviation involving just one change. This follows from the "one-shot deviation principle."

Regularity Condition

Where did we need a regularity condition?
In order to reward punishers you need to have the possibility of phase three payoffs that reward a punisher (but not a deviator).

Variations

- Finite Games when G has multiple equilibria.
- Incomplete information.
- Incomplete Monitoring
- Infinite Horizon Games that Aren't Repeated Games
- Limited Interaction

Key Insight

Folk Theorems work because one can have more than one continuation payoff. This enables the future payoff to depend on today's actions. Hence punishment is possible.

Imperfect Monitoring

Basic Example: Quantity Competition with Noise
This is a simplified overview. Mailath and Samuelson's text (beginning of Chapter 11) has a higher level treatment. Green and Porter (JET, 1984) introduce a basic model.

1. Two firms.
2. Firms select quantity q_i.
3. Stage game payoff, linear Cournot: $\theta[1-(q_1+q_2)]q_i - cq_i$, c is constant marginal cost, θ is demand shock.
4. Firms pick quantities; nature picks θ; firms observe payoff – strictly speaking market price $\theta[1-(q_1+q_2)]$ is observed by both players; θ and opponent's quantity, not public; repeat (with discounting and θ iid).

Analysis

1. Simple static analysis (linear Cournot, with average shock in payoffs).
2. Static equilibrium output larger that joint profit maximizing.
3. Question: If patient players play repeatedly, can they do better than static monopoly?
4. If they can do better, can they do as well as joint profit maximizing?

Routine Computations

Static Duopoly:

$$\theta(1 - 2q_i - q_j) - cq_i = 0 (i \neq j)$$

or

$$q_i = \frac{(\theta - c)}{3\theta} (q_i = 0 \text{ if } c > \theta)$$

$$\pi_i = \left(\frac{\theta - c}{3\theta}\right)^2 \theta$$

Collusion:

$$q_i = \frac{(\theta - c)}{4\theta}$$

$$\pi_i = \left(\frac{\theta - c}{\theta}\right)^2 \frac{\theta}{8}$$

Review

1. Static: Nash equilibrium dominated by collusive behavior.
2. Repetition with patient players can make collusion an equilibrium if θ is known.
3. What if θ is not known?

Some things don't change

1. Can talk about (subgame perfect) equilibrium.
2. Can talk about feasible (expected) stage-game payoffs.
3. Can talk about minmax. (What is it?)
4. Potential to punish exists.

Some things are different

- In complete information game, you know when someone has deviated.
- In imperfect monitoring game, you do not.
- Low profit may be the result of low realization of θ – bad luck – or high output by opponent – cheating.
- If you punish when profit is low, then you run the risk of punishing someone because of bad luck.
- If you do not punish when profit is low, opponent will have incentive to cheat.

Hence folk theorem is in doubt.