Econ 200C - April 2, 2012

Joel Sobel

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INTRODUCTION

Joel Sobel, Room 311. Available after class and by appointment.
Kristy Buzard, Room 123. Available Thursday 11-12 and whatever works best.
Handouts:

1. The outline
2. One problem set.

The Webpage: www.econ.ucsd.edu/ jsobel/200C12:

1. Link on my web page.
2. Will contain announcements, links to handouts, notes, and slides.

IMPORTANT INFORMATION

- I intend to post these slides (for as long as I create them). I urge you to use this as a reason to take fewer notes (instead, think and listen in class).
- Replacement classes: Friday April 6 and Friday 27 (9:40) instead of Wednesday April 11 and Wednesday April 25.

OUTLINE

- Subject: Continuation of Repeated Games, Incomplete information games and applications.
- Unlike 200B Part 1: Cannot summarize class with one model and two basic results.
- Big picture: subtle definitions (learn what a strategy is), techniques, and a few basic models.
- Paternalism: Work as many problems as you can, carefully, and seriously. Try to create interesting problems.
- Requirements: Homework, Midterm
- Miller and I will aggregate grades by averaging them.

Warning

I don't like to use slides.
Advantages:

- It will look like I am prepared.
- You'll have access to the slides.
- I won't need to stand for two hours.
- Maybe you get an advance overview of lecture material.

Disadvantages:

- I might not sustain the energy to prepare notes.
- Typos.
- Tendency to go too fast.

Repeated Games

Relevant Reading: Nageeb's Notes, Chapter 1 Ingredients

1. A game G.
2. A discount factor $\delta \in (0, 1)$.
3. n (either and a positive integer or infinity).

Loosely:
players play G, observe the outcome, play G again, and so on.
Payoffs are the discounted sum of the payoffs from each "stage."

Formal Definition

1. The repeated game has the same player set as G.
2. Strategies:
   - Let $\mathcal{H}_0 = \emptyset$ and $\mathcal{H}_t = S^t$ for $t > 0$.
   - The strategy set for player i: all sequences of functions $f_i^t : \mathcal{H}_{t-1} \to S_i$.
3. The payoff function for the repeated game given strategy profile f is:

$$U_i(f) = (1-\delta^n)^{-1}(1-\delta)\sum_{t=1}^{n}\delta^{t-1}u_i(f^t(h_{t-1}))$$

   where $f^t = (f_1^t, \ldots, f_I^t)$ and h_t is the sequence of histories induced by f $(h_t = h_{t-1} \times f^t(h_{t-1})))$

The interpretation is that $f_i^t(h_{t-1})$ is the action that i takes in the tth repetition following history h_{t-1}. Some literature on "time average" criterion: roughly $\delta = 1$ case (which causes technical problems in infinite horizon games).

Comments

1. Strategy Space is HUGE.
2. $(1-\delta^n)^{-1}(1-\delta)\sum_{t=1}^{n}\delta^{t-1} = 1.$
3. Hence $(1-\delta^n)^{-1}(1-\delta)$ is a normalization factor (designed to make feasible payoff set independent of n).

Definitions

1. Feasible Set $F(G)$: convex hull of $\{u(s) : s \in S\}$.
2. Minmax: $\underline{v}_i^i = \min_{\sigma_{-i} \in \Pi_{j \neq i} \Delta(S_j)} \max_{s_i \in S_i} u_i(s_i, \sigma_{-i})$. (Let $\underline{v}_j^i$ be the payoff that j obtains for strategies that lead to the payoff $\underline{v}_i^i$ for i.)
3. Individually Rational Set of Payoffs: $IR(G) = \{v : v_i \geq \underline{v}_i^i \text{ for all } i\}$.

The set of strictly individually rational payoffs is:

$$SIR(G) = \{v : v_i > \underline{v}_i^i \text{ for all } i\}.$$

The feasible set consists of all possible payoffs that one could obtain from correlated strategies in the given game.
$\underline{v}_i^i$ is a lower bound to the payoff that player i can obtain in a Nash equilibrium. (The definition allows i to best respond to the strategy of the other players, but for the other players to pick the strategy that makes i worse off.)

Largest Possible Equilibrium Payoff Set

The set of equilibrium payoffs must be contained in the set of feasible and individually rational payoffs. This is true for G and for any repeated version of G (with proper normalization).

Subgames

Given any history, h_t, there is a subgame. The subgame is formally identical to the original game (same player set, same strategy sets, same payoff function). Strategies for the original game induce strategies for the subgame. Given a strategy f for the original game, f' defined by $f'(k_s) = f(h_t, k_s)$ is the strategy induced by f. (h_t, k_s) is a $t + s$ period history. Similarly, subgame payoffs are induced. Denote these by $U_i(\cdot \mid h_t)$