Simple Math: Matching Problems

Exercise 1 Construct an example in which there is more than one stable matching. (You only need two boys and two girls to do this.)

Exercise 2 Suppose that the boys all have different favorite girls. How many steps does it take for the algorithm to converge?

Exercise 3 Suppose that the boys have identical preferences. How many steps does it take for the algorithm to converge?

Exercise 4 Suppose preferences are given by the following tables:

BOY	1	2	3	4	5
Adam	Beth	Amy	Diane	Ellen	Cara
Bill	Diane	Beth	Amy	Cara	Ellen
Carl	Beth	Ellen	Cara	Diane	Amy
Dan	Amy	Diane	Cara	Beth	Ellen
Eric	Beth	Diane	Amy	Ellen	Cara

Boys' Preferences

GIRL	1	2	3	4	5
Amy	Eric	Adam	Bill	Dan	Carl
Beth	Carl	Bill	Dan	Adam	Eric
Cara	Bill	Carl	Dan	Eric	Adam
Diane	Adam	Eric	Dan	Carl	Bill
Ellen	Dan	Bill	Eric	Carl	Adam

Girls' Preferences

Find a stable matching using the Gale-Shapley algorithm with boys making proposals. Find a stable matching using the Gale-Shapley algorithm with girls making proposals.

Exercise 5 This exercise shows that stable matchings need not exist if there are not "two sides." Consider the following "roommate" problem. There are four people, Pat, Chris, Dana, and Leslie. They must pair off (each pair will share a two-bed suite). Each has preferences over which of the others they would prefer to have as a roommate. The preferences are:

Leslie: Pat $\succ$ Chris $\succ$ Dana

Chris: Leslie $\succ$ Pat $\succ$ Dana

Pat: Chris $\succ$ Leslie $\succ$ Dana

Dana: Chris $\succ$ Leslie $\succ$ Pat

Show that no stable matching exists. (That is, no matter who you put together, they will always be two potential roommates who are not matched, but prefer each other to their current roommate.)