

Econ 172A, Fall 2012: Midterm A

**Instructions**

- This exam should have your name on it. If it does not, see me or a TA.
- The examination has 3 questions. Answer them all.
- To receive full credit, you must give brief justifications for your answers in Questions 1 and 3. In Question 1 this means that you must state the reasons for all the inferences you draw for complementary slackness. In Question 3 you must provide reasons for your answers. Explanations need not be long, but they should explain why you have answered correctly. No justification is needed for Question 2.
- You may not use books, notes, calculators or other electronic devices.
- If you do not know how to interpret a question, then ask me.

	Score	Possible
I		41
II		49
III		50
Exam Total		140

1. Consider the linear programming problem:

Find  $x_1, x_2, x_3$ , and  $x_4$  to solve  $\mathbf{P}$ :

$$\begin{array}{rllllll} \max & 10x_1 & + & 2x_2 & & + & 5x_4 \\ \text{subject to} & x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & \leq & 10 \\ & x_1 & & & & - & x_3 & + & 2x_4 & \leq & 20 \\ & x_1 & + & x_2 & & & & - & x_4 & \leq & 5 \\ & & & & & x & \geq & & & & 0 \end{array}$$

You must provide justifications for your answers to the questions below. In particular, say what you need to do to check for feasibility and explain the basis for your inferences in part (c).

- (a) Write the dual of the problem  $\mathbf{P}$ .
- (b) Verify that  $(x_1, x_2, x_3, x_4) = (6, 0, 0, 1)$  is feasible for  $\mathbf{P}$ .
- (c) Assuming that  $(6, 0, 0, 1)$  is a solution to  $\mathbf{P}$ , use Complementary Slackness to determine a candidate solution to the dual.
- (d) Is  $(6, 0, 0, 1)$  a solution to  $\mathbf{P}$ ? Explain.
- (e) Do you have enough information to find the solution to the dual of  $\mathbf{P}$ ? If so, find it.

2. I solved a linear programming problem written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0.$$

Attached find the Excel Answer and Sensitivity report. (I deleted some irrelevant information.) In these reports, I replaced several values with letters ((a) through (k)). Using the information in the table, replace as many of these letters with the correct information. You need not justify these answers (simply write the answers in the appropriate spaces, next to the letter). If you do not have enough information to figure out one or more of the values, write "NOT ENOUGH INFORMATION" next to the letter.

In addition to completing the tables, please answer the following questions. Once again, if you do not have sufficient information to answer a question, write "not enough information."

- (a) How many variables are in the original problem (primal)?
  
- (b) How many variables are in the dual?
  
- (c) What is the objective function of the primal?
  
- (d) What is the objective function of the dual?
  
- (e) What is the value of the primal?
  
- (f) What is the value of the dual?
  
- (g) What is the solution to the primal?
  
- (h) What is the solution to the dual?

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 Question 2a, data

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$E\$8	x1	4	(a)	15	(b)	3
\$F\$8	x2	12	0	6	1.5	0.423077
\$G\$8	x3	0	(c)	9	15	(d)
\$H\$8	x4	0	-0.55	2	(e)	1.00E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$I\$13	#1 LHS	(f)	(g)	20	4	4
\$I\$15	#3 LHS	28	(h)	70	(i)	(j)
\$I\$14	#2 LHS	24	3	(k)	6	4

Place Answers Below

- a
- b
- c
- d
- e
- f
- g
- h
- i
- j
- k

3. A drug company manufactures two kinds of sleeping pill: Pill 1 and Pill 2. The pills contain a mixture of two different chemical,  $A$  and  $B$ . By weight, the Pill 1 must contain at least 65% Chemical  $A$  and Pill 2 must contain at least 55% Chemical  $A$ . There are two different ways to produce Chemicals  $A$  and  $B$ . Operating Process  $P$  for one hour requires 7 ounces of a raw material and 2 hours of labor time (because it must be supervised by 2 workers). Operating Process  $Q$  for one hour requires 5 ounces of a raw material and 3 hours of labor time. Operating Process  $P$  for one hour produces three ounces of each chemical. Operating Process  $Q$  for one hour produces 3 ounces of Chemical  $A$  and one ounce of Chemical  $B$ . There are  $L > 0$  hours of labor and  $R > 0$  ounces of raw material are available. The company earns a profit from Pill  $i$  of  $\pi_i \geq 0$  dollars per ounce.

Here is a potential formulation for the problem.

Introduce the variables,  $z_Q$ ,  $z_P$ ,  $x_1$ , and  $x_2$ .  $z_Q$  represents the number of hours Process  $Q$  is operated.  $z_P$  represents the number of hours Process  $P$  is operated.  $x_i$  represents the number of ounces of Pill  $i$  produced ( $i = 1$  or  $2$ ).

The problem becomes: Find  $z_Q$ ,  $z_P$ ,  $x_1$ , and  $x_2$  to solve:

$\max \pi_1 x_1 + \pi_2 x_2$  subject to:

- (1)  $2z_P + 3z_Q \leq L$
- (2)  $7z_P + 5z_Q \leq R$
- (3)  $x_1 + x_2 \leq 6z_P + 4z_Q$
- (4)  $.65x_1 + .55x_2 \leq 3z_P + 3z_Q$
- (5)  $x_1, x_2, z_P, z_Q \geq 0$

Answer the follows questions as completely as possible.

- (a) Interpret constraint (3). That is, explain in words what the constraint says and how it relates to the problem. (Three short sentences suffice.)
  
- (b) Explain why the problem has a solution (that is, demonstrate that it is neither infeasible nor unbounded).
  
- (c) Suppose that  $\pi_1 < \pi_2$ . Will there necessarily exist a solution to the problem in which  $x_1 = 0$ ?
  
- (d) Suppose that  $\pi_1 > \pi_2$ . Will there necessarily exist a solution to the problem in which  $x_2 = 0$ ?

- (e) Suppose that both prices double (so that the company earns  $2\pi_i$  dollars per ounce of Pill  $i$  produced). What happens to the solution to the problem and the company's maximum profit?
- (f) Suppose that both prices increase by \$1 (so that the company earns  $\pi_i+1$  dollars per ounce of Pill  $i$  produced). What happens to the solution to the problem and the company's maximum profit?
- (g) Suppose that  $L$  increases. Under what conditions will the solution to the problem remain unchanged?
- (h) Let  $y_{1A}$  describe the amount of Chemical  $A$  used in Pill 1. Write down a set of inequalities (in terms of  $x_1$ ,  $x_2$ ,  $z_P$  and  $z_Q$ ) that constrain the value of  $y_{1A}$ . (Your answer should be in the form of a set of linear inequalities or equations using the four variables and the information above. For example, if you think that the amount of Chemical  $A$  used in Pill 1 is 5 ounces, write  $y_{1A} = 5$ . If you think that the amount of Chemical  $A$  used in Pill 1 is less than the total number of Pill 1 produced and at least 20% of the number of Pill 1 produced, write  $.2x_1 \leq y_{1A} \leq x_1$ .)