Econ 172A - Slides from Lecture 9

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Announcements

- Important: Midterm seating assignments. Posted.
- Corrected Answers to Quiz 1 posted.
- Midterm on November 1, 2012.
- Problems for Midterm:
    1. 2004: Midterm #1: 2, 3; Midterm #2. Problem Set 1, Problem Set 2 - 1,2. Final: 2, 3, 5.
    2. 2007: Midterm. Problem Sets 1 and 2.
    3. 2008: Midterm. Problem Sets 1 and 2. Final: 5, 7 a, b.
    4. 2010: Midterm. Quizzes 1, 2. Final 1 a,b; 4

Material for Midterm

- Problem Formulation
- Graphing
- Duality (all topics)
- Interpreting Excel Spreadsheets
  I can ask you to “fill in the blanks” in a sensitivity table.

This means: material through today’s lecture (but Tuesday’s lecture practice useful skills for the exam).

Reduced Cost

- Reduced cost of a variable: smallest change in the objective function coefficient needed to arrive at a solution in which the variable takes on a positive value when you solve the problem.
- Meaning??
- Algebraic Definition (Non-basic): The reduced cost of a non-basic variable is the negative of the allowable increase (that is, if you change the coefficient of x_1 by -7, then you arrive at a problem in which x_1 takes on a positive value in the solution).
- Economic Interpretation: The allowable increase in the coefficient is 7.
- The reduced cost of a basic variable is always zero.
- Economic Interpretation: You need not change the objective function at all to make the variable positive.

More Reduced Cost

- You can figure out reduced costs from the other information in the table: If the final value is positive, then the reduced cost is zero.
- Warning (for experts, not exam material): The previous statement fails in "degenerate" cases. For details, see me.
- If the final value of a variable is zero, then the reduced cost is negative one times the allowable increase.
- The reduced cost of a variable is also the amount of slack in the dual constraint associated with the variable.
- With this interpretation, complementary slackness implies that if a variable that takes on a positive value in the solution, then its reduced cost is zero.

Sensitivity Information on Constraints

- Second sensitivity table.
- The cell column identifies the location of the left-hand side of a constraint.
- The name column gives its name (if any).
- The final value is the value of the left-hand side when you plug in the final values for the variables.
- The shadow price is the dual variable associated with the constraint.
- The constraint R.H. side is the right hand side of the constraint.
- The allowable increase tells you by how much you can increase the right-hand side of the constraint without changing the basis.
- The allowable decrease tells you by how much you can decrease the right-hand side of the constraint without changing the basis.

Complementary Slackness Again

- Complementary Slackness guarantees a relationship between the columns in the constraint table.
- The difference between the "Constraint Right-Hand Side" column and the "Final Value" column is the slack.
- For example, the slack for the three constraints is 0 ($= 12 - 12$), 37 ($= 7 - (-30)$), and 0 ($= 10 - 10$), respectively.
- Complementary Slackness says that if there is slack in the constraint then the associated dual variable is zero.
- Hence CS tells us that the second dual variable must be zero.

- You can figure out information on allowable changes from other information in the table.
- The allowable increase and decrease of non-binding variables can be computed knowing final value and right-hand side constant.
- If a constraint is not binding, then adding more of the resource is not going to change your solution.
- Hence the allowable increase of a resource is infinite for a non-binding constraint.
- Similarly: the allowable increase of a resource is infinite for a constraint with slack.
- So in example allowable increase of the second constraint is infinite.

- Also: The allowable decrease of a non-binding constraint is equal to the slack in the constraint.
- Hence the allowable decrease in the second constraint is 37.
- If you decrease the right-hand side of the second constraint from its original value (7) to anything greater than -30 you do not change the optimal basis.
- The only part of the solution that changes is that the value of the slack variable for this constraint.
- If you solve an LP and find that a constraint is not binding, then you can remove all of the unused (slack) portion of the resource associated with this constraint and not change the solution to the problem.

Allowable Increase and Decrease

- First constraint. If the right-hand side of the first constraint is between 10 (original value 12 minus allowable decrease 2) and infinity, then the basis of the problem does not change.
- Solution usually does change.
- Saying that the basis does not change means that the variables that were zero in the original solution continue to be zero in the new problem (with the right-hand side of the constraint changed).
- When the amount of available resource changes, necessarily the values of the other variables change.
- In diet problem, getting requiring more nutrient of something that you had supplied exactly before leads you to buy more food.
- In production problem, getting more of a scarce ingredient allows you to produce more.
- Changes within the allowable range for a binding constraint's RHS doesn't change the positive variables in the solution, but

Third Constraint

- The values for the allowable increase and allowable decrease say that the basis that is optimal for the original problem (when the right-hand side of the third constraint is equal to 10) remains obtain provided that the right-hand side constant in this constraint is between -2.3333 and 12.
- Suppose that your LP involves four production processes and uses three basic ingredients.
- Call the ingredients land, labor, and capital.
- The outputs vary use different combinations of the ingredients.
- Maybe they are growing fruit (using lots of land and labor), cleaning bathrooms (using lots of labor), making cars (using lots of labor and and a bit of capital), and making computers (using lots of capital).
- For the initial specification of available resources, you find that your want to grow fruit and make cars.

- If you get an increase in the amount of capital, you may wish to shift into building computers instead of cars.
- If you experience a decrease in the amount of capital, you may wish to shift away from building cars and into cleaning bathrooms instead.

Duality

- The "Adjustable Cells" table and the "Constraints" table provide the same information.
- Dual variables correspond to primal constraints.
- Primal variables correspond to dual constraints.
- The "Adjustable Cells" table tells you how sensitive primal variables and dual constraints are to changes in the primal objective function.
- The "Constraints" table tells you how sensitive dual variables and primal constraints are to changes in the dual objective function (right-hand side constants in the primal).

Example

Simple formulation exercise.

- A furniture company that makes tables and chairs.
- A table requires 40 board feet of wood.
- A chair requires 30 board feet of wood.
- Wood costs $1 per board foot and 40,000 board feet of wood are available.
- It takes 2 hours of labor to make an unfinished table or an unfinished chair.
- 3 more hours of labor will turn an unfinished table into a finished table.
- 2 more hours of labor will turn an unfinished chair into a finished chair.
- There are 6000 hours of labor available.
- No need to pay for this labor.

Prices

The prices of output are given in the table below:

Product	Price
Unfinished Table	$70
Finished Table	$140
Unfinished Chair	$60
Finished Chair	$110

Formulation

- Objective: Describe the production plans that the firm can use to maximize its profits.
- Variables: Number of finished and unfinished tables and chairs.
- Let T_F and T_U be the number of finished and unfinished tables.
- Let C_F and C_U be the number of finished and unfinished chairs.
- Revenue:

$$70T_U + 140T_F + 60C_U + 110C_F,$$

.

- Cost is $40T_U + 40T_F + 30C_U + 30C_F$ (because lumber costs \$1 per board foot).
- Profit (Revenue - Cost): $30T_U + 100T_F + 30C_U + 80C_F$.

- The constraints are:
  1. $40T_U + 40T_F + 30C_U + 30C_F \leq 40000$.
  2. $2T_U + 5T_F + 2C_U + 4C_F \leq 6000$.
- The first constraint says that the amount of lumber used is no more than what is available.
- The second constraint states that the amount of labor used is no more than what is available.

Solution

- Excel finds the answer to the problem to be to construct only finished chairs (1333.333).
- It is crazy to produce $\frac{1}{3}$ chair.
- This means that the LP formulation is not literally correct.
- Now: We pretend that fractional chairs are ok.
- Later: We will introduce methods to deal with the constraint that variables should be integers.
- The profit is \$106,666.67.