

Econ 172A - Slides from Lecture 5

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Announcements

- ▶ Quiz 2 on October 25, 2012.
- ▶ Problems: Problems on current material: Problem Set 2, 2004: #1, #4; 2007: #1; 2008: #2.
Midterm 1, 2004: all; Midterm 2007: #1, 4; Midterm 1, 2008: #2

Theorem (Complementary Slackness)

Assume problem (P) has a solution x^* and problem (D) has a solution y^* .

1. If $x_j^* > 0$, then the j th constraint in (D) is binding.
2. If the j th constraint in (D) is not binding, then $x_j^* = 0$.
3. If $y_i^* > 0$, then the i th constraint in (P) is binding.
4. If the i th constraint in (P) is not binding, then $y_i^* = 0$.

Algebra

Step 1, check feasibility.

- ▶ First constraint: $0 + 10.4 + 0 + 1.6 = 12$. Binding. Tells you nothing about y_1 .
- ▶ Second constraint: $0 - 31.2 + 0 + 1.2 = -30$. Not binding (but satisfied). $y_2 = 0$.
- ▶ Third constraint: $0 + 10.4 + 0 - .4 = 10$. Binding. Tells you nothing about y_3 .

So x satisfies constraints.

Step 2, implications of positive variables.

1. $x_1 = 0$ tells you nothing.
2. $x_2 > 0$ tells you second dual constraint binds at solution:

$$y_1 - 3y_2 + y_3 = 4$$

3. $x_3 = 0$ tells you nothing.
4. $x_4 > 0$ tells you fourth dual constraint binds:

$$4y_1 + 3y_2 - y_3 = 1.$$

So solution to dual must satisfy: $y_2 = 0$, $y_1 + y_3 = 4$, and $4y_1 - y_3 = 1$.

This means that candidate for solution to (D) is $y = (1, 0, 3)$

Is it a solution?

Need to check constraints we ignored:

1. Non negativity
2. Dual Constraints 1 and 3.

These constraints do hold.

It is a solution.

(Check that values are equal.)

What Happened?

1. Begin with a “guess” for Primal.
2. Check feasibility. If not, stop. It cannot be a solution.
3. If so, note positive variables (implies binding dual constraints) and constraints with slack (implies zero dual values).
4. Use non-zero dual variables to solve the equations in dual. (Typically equal number of equations and unknowns.)
5. Check feasibility of this solution (are variables nonnegative? Are the dual constraints corresponding to zero primal variables satisfied?) If yes, then you have solutions to both Primal and Dual. If not, original guess is not a solution.