Econ 172A - Slides from Lecture 2

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Announcements

1. Sections this evening (York 226, 8-9 or 9-10).
2. Podcasts available when I remember to use microphone.
3. Textbook on reserve at SSH Library.
4. Look at Supplementary Formulation Problems
5. Updated notes posted.

PROBLEM FORMULATION

1. Problem Formulation is the process of translating a natural-language problem into math.
2. I can't teach this, but maybe I can illustrate it

DIET PROBLEM

1. Given:
   - A list of different foods.
   - A list of different nutrients.
   - The unit price of each food.
   - The minimum daily requirement of each nutrient.
   - The nutrient contribution of each food.
2. Find the cheapest way to minimize all nutritional requirements.

BASIC DATA

1. n different kinds of food.
2. p_j price per unit of jth food.
3. m different nutrients.
4. nutritional requirement of Nutrient i is c_i.
5. A is technology (a_{ij} is the amount of the ith nutrient in one unit of the jth food).

INFORMALLY

1. Foods: lettuce, peanut butter, bread, apple juice. F_j, the jth food, is one of these.
2. Nutrients: Vitamin B12, iron, calcium, ... N_i, the ith nutrient, is one of these.
3. Everything has units:
   - 3.1 prices "dollars per unit of food"
   - 3.2 nutrient requirements: "units of nutrient."
   - 3.3 a_{ij}: " units of nutrient per unit of food"

Step 1: Identify Variables.

What are you looking for?

- You are looking for amounts of food.
- Variables are quantities of each of the n foods.
- These are unknowns and need names.
- Let x_j be the number of units of food j purchased.
- You want to find $x = (x_1, \ldots, x_n)$.

IMPORTANT

- The problem statement typically identify the variables. That is, it doesn't say:
  "Your job is to find x, where x_j is the quantity of Food j."
- You must not only define variables, you must specify the units. (Here, it is uninteresting: x_j is the number of units of F_j.)

Step 2: Write Down the Objective Function.

What are you trying to do?

Minimize cost.
Minimize cost of the food that you buy.
If you buy x you pay

$$p_1x_1 + \cdots + p_jx_j + \cdots + p_nx_n = \sum_{j=1}^{n} p_jx_j = p \cdot x. \tag{1}$$

(1) is the objective function. That is, you want to find x to $\min p \cdot x$.

NOTICE LINEARITY ASSUMPTION

Step 3: Write Down the Constraints.

- The constraints are that you satisfy nutritional requirements.
- You need to buy enough food to supply all nutrients in (at least) the recommended amounts.
- How much nutrient i do you need? c_i.
- How much of this nutrient is supplied when you have x? Next page.

Writing the Constraints

- You buy x_1 units of the first food.
- You obtain $a_{i1}x_1$ units of the ith nutrient coming from the first food.
- Notice: product is in units of nutrient.
- How much nutrient i do you get from x?

$$a_{i1}x_1 + \cdots + a_{ij}x_j + \cdots + a_{in}x_n = \sum_{j=1}^{n} a_{ij}x_j. \tag{2}$$

- The constraint:

$$a_{i1}x_1 + \cdots + a_{ij}x_j + \cdots + a_{in}x_n = \sum_{j=1}^{n} a_{ij}x_j \geq c_i \tag{3}$$

  describes the ith nutritional constraint.
- The entire problem imposes such a constraint for each nutrient. That is we need an inequality for $i = 1, \ldots, m$.

Cleaning Up Constraints

$$Ax \geq c$$

summarizes all m constraints.

Reflect on Linearity Assumptions Implicit in Constraints

Nonnegativity

Implicit in problem:
$x \geq 0$.

Step 4: Write Down the Entire Problem.

The problem is to find x to solve:

$$\min p \cdot x \text{ subject to } Ax \geq c \text{ and } x \geq 0.$$

In practice, you will be given values for the parameters of the problem (A, p, and c) and then would go ahead and try to find a numerical solution.
http://www.zweigmedia.com/RealWorld/dietProblem/diet.html