

## Econ 172A, Fall 2010: Quiz III

**Grading Notes:** 40 points possible, median 30. (5 points for each question.)

A company produces two products:  $A$  and  $B$ . Product  $A$  sells for \$11 per unit and product  $B$  sells for \$23 per unit. Producing a unit of product  $A$  requires 2 hours on assembly line 1 and 1 unit of raw material. Producing a unit of product  $B$  requires 2 units of raw material, 1 unit of  $A$ , and 2 hours on assembly line 2. On line 1,  $L_1$  hours of time are available, and  $L_2$  hours are available on line 2. A unit of raw material can be bought (for \$5 a unit) or produced (at no cost) using 2 hours of time on line 1.

I solved the problem using the formulation below.

Definition of variables:

$x_A$  number of product  $A$  sold

$x_B$  number of product  $B$  sold

$x_R$  units of raw material purchased

$x_P$  units of raw material produced

objective function:  $\max 11x_A + 23x_B - 5x_R$

Constraints:

Assembly line 1 constraint:  $2x_A + 2x_B + 2x_P \leq L_1$

Assembly line 2 constraint:  $2x_B \leq L_2$

Raw material:  $x_A + 3x_B - x_R - x_P \leq 0$

Answer the following questions as completely as possible (if you have insufficient information, say so). You should state whether the proposed change is inside the allowable range and how the indicated quantity or quantities (solution or value) changes. If you do not know exactly how a quantity changes, say as much as you can (for example, “it goes up,” or “it goes down by at least 40,” or “it does not go up by more than 10.”)

1. What is  $L_1$ ?

Form 1: 1300; Form 2: 500; Form 3: 1000

2. What is the solution?

Form 1:  $(x_A, x_B, x_R, x_P) = (400, 250, 1150, 0)$ ; Form 2:  $(x_A, x_B, x_R, x_P) = (150, 100, 450, 0)$ ;

Form 3:  $(x_A, x_B, x_R, x_P) = (300, 200, 900, 0)$

3. What is the value of the problem if the amount of Assembly line 2 time doubles (from  $L_2$  to  $2L_2$ )?

Form 1: Inside allowable range. Goes up by 500 (the amount of increase is 500 and shadow price is 1).

Form 2: Inside allowable range. Goes up by 200 (the amount of increase is 200 and shadow price is 1)

Form 3: Inside allowable range. Goes up by 400 (the amount of increase is 400 and shadow price is 1).

4. How does the solution and value change if Product  $A$ 's sale price increases by 1 (to \$12)?

Form 1: Within allowable range. Same solution. Value increases by 400.

Form 2: Within allowable range. Same solution. Value increases by 150.

Form 3: Within allowable range. Same solution. Value increases by 300.

5. How does the solution and value change if the price of raw material increases by 1 (to \$6 per unit)?

Form 1: Outside of allowable range. Value goes down by at least \$1150.

Form 2: Outside of allowable range. Value goes down by at least \$450.

Form 3: Outside of allowable range. Value goes down by at least \$900.

6. How does the solution and value change if Product  $B$ 's sale price decreases by 3 (to \$20).
- Form 1: Outside of allowable range. Two units are in allowable range. The value goes down by at least 250 times 2. Solution will change and value will go down by no more than 3 times 250 (this is how the value would change if you didn't change solution).
- Form 2: Outside of allowable range. Two units are in allowable range. The value goes down by at least 100 times 2. Solution will change and value will go down by no more than 3 times 100 (this is how the value would change if you didn't change solution).
- Form 3: Outside of allowable range. Two units are in allowable range. The value goes down by at least 200 times 2. Solution will change and value will go down by no more than 3 times 200 (this is how the value would change if you didn't change solution).
7. Suppose that the company could use one hour of Assembly line #1 time and one hour of Assembly Line # 2 time and 4 units of raw material to produce one unit of a new product,  $C$ . Would it be profitable to produce  $C$  if it could be sold for \$25 per unit?
- All forms: Yes:  $25 > 3 + 1 + 5 \times 4$
8. How would the value change if the company faced an additional constraint that required the total amount of machine time used to be less than 1200 hours?
- Form 1: Currently you use all available time, 1800 hours. So solution and value must go down. (No need to say this: You can tell that it will go down by at least 500.)
- Form 2: Currently you use all available time, 700 hours. So solution and value stay the same (new constraint isn't binding).
- Form 3: Currently you use all available time, 1400 hours. So solution and value must go down. (That answer is sufficient. You can tell that it will go down by 200.)