## Econ 172A, Fall 2010: Quiz 2 Answers

Below  $(D'_t)$  refers to the dual of (P') and  $(D_t)$  to the Dual of (P). [The novelty of the question is that what I call (D) or D') are not duals of the given problems because of the omitted non-negativity constraint.]

## Form 1 answers:

- 1. True: Feasible set is larger for (P').
- 2. False:  $(D_t)$  and  $(D'_t)$  may be infeasible and (P') could be unbounded. Example: A = -1, b = b' = -1, c = 1. (D) has solution (y = -1).  $(D_t)$  and  $(D'_t)$  are not feasible. (P) and (P') are unbounded.
- 3. True: (D) not feasible implies that  $(D_t)$  is not feasible (more constraints) implies that  $(D'_t)$  is not feasible (same feasible set as  $(D_t)$ ) implies that (P') has no solution.
- 4. False: Among other things, wrong objective function.
- 5. False: (P) may be unbounded. Try  $\max 2x_1 x_2$  subject to  $x_1 x_2 \le 0$ ,  $x_1, x_2 \ge 0$ . Check that (D) is not feasible.
- 6. True: Bigger feasible set for (P').
- 7. False: This would be true without the non-negativity constraint, but not here. Example: (P) is  $\max x_2$  subject to  $x_1 + x_2 \le 2$  and  $x_1 x_2 \le 0$  and  $x \ge 0$ . The solution is  $x^* = (0, 2)$ ,  $c \cdot x^* = 2$ . (D) is  $\min 2y_1$  subject to  $y_1 + y_2 \ge 0$ ,  $y_1 y_2 \ge 1$ . Solution  $y^* = (.5, -.5)$ . The equation  $c \cdot x^* = y^*Ax^*$  does not hold.
- 8. False: (D) could be unbounded. Example: b = b' = -1, c = 0, A = 1.

## Form 2 Answers

- 1. False: Among other things, the objective function is not right.
- 2. False: May have (D) unbounded and (P') not feasible.
- 3. True: Same feasible set.
- 4. False:  $(D'_t)$  may be infeasible (so that (P') has no solution). (For example, constraint set of (D) is  $-y \ge 1$ .)
- 5. False: Lowering c to c' may make  $(D'_t)$  feasible.
- 6. False: (P) may be unbounded.
- 7. False: Less than or equal.
- 8. False: See Form 1, 7.

## Form 3 Answers

- 1. False: See Form 1, 7.
- 2. False: (D') lacks non-negativity constraints.
- 3. False:  $(D'_t)$  may not be feasible.
- 4. False: (P') has more stringent constraints. It may be infeasible.
- 5. False:  $(D'_t)$  may be infeasible and then (P) will be unbounded.
- 6. True: (D') not feasible means  $(D'_t)$  and  $(D_t)$  are not feasible, so (P) has no solution.
- 7. False: Among other things, (P') could be unbounded.
- 8. True: (P) has larger feasible set so solution to (P') is feasible for (P).