Econ 172A, F2008: Midterm Examination II-A

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.
2. The examination has 3 questions. Answer them all.
3. You may not use calculators, books, or notes during this exam.
4. If you do not know how to interpret a question, then ask me.
5. You must justify your answers to the first two questions. No justification is needed on the last question.
6. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		30
II		27
III		24
Exam Total		81

1. W Oil has 5,000 barrels of crude oil type 1 and 10,000 barrels of crude oil type 2 available. W sells gasoline and heating oil. These products are blends of the two crude oils. Each barrel of crude oil 1 has quality level 10 and each barrel of crude oil 2 has quality level 5. Gasoline must have an average level of at least 8 while heating oil must have an average quality level of at least 6. Gasoline sells for $25 per barrel, and heating oil sells for $20 per barrel. Demand for heating oil and gasoline is unlimited. I have formulated a linear programming problem that describes W's revenue-maximizing problem below.

   Define x_{ij} to be the amount of oil type i used in blend j, $i = 1$ or 2; $j = G$ (for gasoline) or O (for heating oil). So in particular, x_{1G} is the amount of crude oil type 1 put in the blend for gasoline and $x_{1G} + x_{2G}$ is the total amount of gasoline produced.

$$\begin{array}{llllllllll}
\max & 25x_{1G} & + & 20x_{1O} & + & 25x_{2G} & + & 20x_{2O} & & \\
\text{subject to} & x_{1G} & + & x_{1O} & & & & & \leq & 5000 \\
 & & & & & x_{2G} & + & x_{2O} & \leq & 10000 \\
 & -2x_{1G} & & & + & 3x_{2G} & & & \leq & 0 \\
 & & & -4x_{1O} & & & + & x_{2O} & \leq & 0
\end{array}$$

   and $x \geq 0$. Use the attached sensitivity table to answer the questions that follow. You may not have enough information to answer each part completely, but you should provide as much relevant information as possible. (For example, if you do not know the new value, but you know that the value must decrease, or must decrease by at least 100, then you should say this.) Justify your answers.

   (a) Explain the third constraint ($-2x_{1G} + 3x_{2G} \leq 0$).

   (b) What is the solution to the problem and the associated value?

   (c) Write the dual of the problem.

   (d) What is the solution to the dual and the associated value?

   (e) How would the value of the problem change if there were 8,000 barrels of Crude oil 1 available (instead of 5,000)?

(f) How would the value of the problem change if there were 2,000 barrels of Crude oil 1 available (instead of 5,000)?

(g) How would the value of the problem change if the price of Gasoline went up to \$30/barrel (from 25)?

(h) How would the value of the problem change if the average quality of Gasoline had to be 8.5 (instead of 8)?

(i) The government enforces average quality standards by monitoring the mixtures. It assigns ten points for each barrel of crude 1 in the mixture and five points for each barrel of crude 2 in the mixture. It then requires that the total number of points in the gasoline mixture be at least eight times the number of barrels sold (and the total number of points in the heating oil mixture be at least six times the number of barrels sold). Suppose that the government regulator gives the company 100 bonus points in the production of gasoline. That is, instead of requiring that the average quality of gasoline be at least eight, it requires that the total number of points in gasoline plus the 100 bonus points is at least eight times the number of barrels of gasoline sold. How would the value of the problem change?

(j) What happens to profits if the total amount of crude oil (both 1 and 2) decreases by 90% (so instead of having 5,000 and 10,000 barrels of crude oil 1 and 2 available, you have 500 and 1,000 barrels available)?

	A	B	C	D	E	F	G	H	I	J	K
1	**Econ 172A**										
2	Joel Sobel										
3											
4	Objective Function Coefficients				c_1G	c_1O	c_2G	c_2O			
5					25	20	25	20			
6											
7	Variables				x_1G	x_1O	x_2G	x_2O			
8					3000	2000	2000	8000			
9											
10											
11											
12	Resource Constraints				ai1	ai2	ai3	ai4	LHS		bi
13				Crude 1	1	1	0	0	5000		5000
14				Crude 2	0	0	1	1	10000		10000
15				Constraint 3	-2	0	3	0	0		0
16				Constraint 4	0	-4	0	1	0		0
17											
18											
19	Value	325000									

Microsoft Excel 11.5 Sensitivity Report
Worksheet: [Workbook2]PS #2- 1
Report Created: 11/9/2008 7:26:38 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
E8	x_1G	3000	0	25	58.33333333	8.333333333
F8	x_1O	2000	0	20	8.333333333	58.33333333
G8	x_2G	2000	0	25	87.5	6.25
H8	x_2O	8000	0	20	6.25	14.58333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I13	Crude 1 LHS	5000	30	5000	10000	2500
I15	Constraint 3 LHS	0	2.5	0	20000	5000
I14	Crude 2 LHS	10000	17.5	10000	10000	6666.666667
I16	Constraint 4 LHS	0	2.5	0	6666.666667	10000

2. I solved a version of a linear programming problem using Excel. I attach the answer report and the sensitivity report from Excel. In these reports, I replaced several values with question marks (???). Your job is to replace these question marks with the correct information. I have not given you enough information to reconstruct the problem. You should fill in the missing values using your knowledge of Excel, duality theory, and complementary slackness. You may not have sufficient information to complete the table. If you cannot determine some of the missing numbers, then say so. If you can fill in a value, then explain what permitted you to do so.

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Fall 2008
Problem 2

Adjustable Cells

	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
	x_1	60	???	5	1	???
	x_2	15	???	8	???	1.333333333
	x_3	???	-4	4	???	???

Constraints

	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
	LHS_1	90	???	220	???	???
	LHS_2	???	???	480	???	120
	LHS_3	112.5	???	3000	???	???
	LHS_4	-60	1	???	60	20

3. For each of the statements below indicate whether the statement is always **TRUE**, by circling "TRUE" otherwise circle "FALSE." No justification is required.

The statements refer to the linear programming problem (LP) written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and an associated integer programming problem (IP):

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0, x \text{ integer}$$

c is a vector with n components, b is a vector with m components, and A is a matrix with m rows and n columns. Assume that both problems are feasible.

TRUE **FALSE** If the value for the (LP) is an integer, then the value of (LP) is equal to the value of (IP).

TRUE **FALSE** If a non-integer solution is feasible for (LP), then the integer solution obtained by rounding down all of the variables to the next lowest integer is feasible for (IP).

TRUE **FALSE** The feasible set for (LP) contains the feasible set for (IP).

TRUE **FALSE** If every x that is feasible for (LP) satisfies $x_i \leq 10$ for $i = 1, 2, \ldots n$, then there are at most 20^n feasible points that are feasible for (IP).

TRUE **FALSE** If the value of the problem:

$$\min c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

is equal to -1.5, then the value of (IP) is greater than or equal to -1.

TRUE **FALSE** If b, c, and all of the entries in A are integers, then the solution of (IP) is a solution to (LP).