Econ 172A, F2008: Midterm Examination I-A

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.

2. The examination has 3 questions. Answer them all.

3. You may not use calculators, books, or notes during this exam.

4. If you do not know how to interpret a question, then ask me.

5. You must justify your answers.

6. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		31
II		25
III		25
Exam Total		81

1. Consider the following linear programming problem. Find x_1 and x_2 to solve:

$$\max x_0$$

$$
\begin{array}{rcrcrcr}
\text{subject to} & 5x_1 & + & x_2 & \leq & 45 \\
& x_1 & - & x_2 & \geq & 5 \\
& -x_1 & + & 3x_2 & \leq & -5
\end{array}
$$

(a) Graphically represent the feasible set of this problem. Label all important points and edges.

(b) Graphically solve the problem for the following values of x_0 below. In each case, using the graph from part a, identify the solution (explain why the graph tells you that you have a solution); write down the values for x_1 and x_2 that solve the problem; write down the value of the problem. If the solution to the problem is not unique, then give two solutions. If the solution does not exist, then explain why it does not exist.

   i. $x_0 = x_1 + x_2$.

   ii. $x_0 = x_2$.

   iii. $x_0 = 2x_1 + x_2$.

(c) Which of the following points can be a solution to a linear programming problem for some (linear) choice of x_0 and the constraint set given above?

    i. $(x_1, x_2) = (7.5, 2.5)$.

    ii. $(x_1, x_2) = (5, 0)$.

    iii. $(x_1, x_2) = (9, 0)$.

    iv. $(x_1, x_2) = (0, 15)$.

    v. $(x_1, x_2) = (6, -6)$.

(d) Which of the points in Part (c) can be a **unique** solution to a linear programming problem for some (linear) choice of x_0 and the constraint set given above? Justify your answers.

(e) Is it possible to find a linear objective function x_0 such that the linear programming problem defined above is not feasible? If so, give an example. If not, explain why it is not possible.

(f) Is it possible to find a linear objective function x_0 such that the linear programming problem defined above is feasible, but has no solution. If so, give an example. If not, explain why it is not possible.

2. Consider the linear programming problem:

$$\begin{array}{llrlrlrlrcl}
\text{max} & 2x_1 & + & 4x_2 & - & 6x_3 & + & 5x_4 & & \\
\text{subject to} & x_1 & + & 4x_2 & + & 8x_3 & - & 2x_4 & \le & 2 \\
& -x_1 & + & 2x_2 & + & 4x_3 & + & 3x_4 & \le & 1 \\
& & & & & & & x & \ge & 0
\end{array}$$

(a) Write the dual of the problem.

(b) Show that $x^* = (x_1^*, x_2^*, x_3^*, x_4^*) = (8, 0, 0, 3)$ is feasible for the original problem.

(c) Use complementary slackness to determine whether $(8, 0, 0, 3)$ is a solution to the original problem.

(d) If $(8, 0, 0, 3)$ is a solution to the original problem, find a solution to the dual. If $(8, 0, 0, 3)$ is not a solution to the original problem, find $x = (x_1, x_2, x_3, x_4)$, such that $x \ne x^*$; x is feasible for the original problem; and x yields a higher value of the objective function than x^* (that is, $2x_1 + 4x_2 - 6x_3 + 5x_4 > 2x_1^* + 4x_2^* - 6x_3^* + 5x_4^*$).

4

3. A local company can produce three products, A, B, and C. The company can sell up to 3000 units of Product A, up to 2000 units of Product B, and up to 2000 units of Product C. Each unit of Product C uses 2 units of A and 3 units of B and also requires an expenditure of \$5 (production costs). Products A and B can be produced from either Process I or Process II (or combinations of these two processes). In Process I the Company can produce two units of A and three units of B for \$6. In Process II, the company can produce one unit of A and two units of B for \$4. The unit prices for the products are \$5 for A, \$4 for B, and \$25 for C. The quality levels of each product are: A, 8; B, 7; C, 6. The average quality level of the units sold must be at least 7.

Use the following definitions: Let x_i be the number of units of product i sold for $i = A, B, C$. [Note: Since A and B are inputs to the process that produces C, the company must produce more than x_A units of A and x_B units of B if it wishes to produce a positive quantity of C.] Let L_j be the level Process j is operated, for $j = I$ and II. (So, for example, if $L_I = 1$ the company is producing two units of A and three units of B for \$6. If $L_{II} = 3$, then the company is producing three units of A and six units of B for \$12.) Use the variables x_A, x_B, x_C, L_I, and L_{II} to answer the problems below.

(a) Write a constraint that guarantees that the average quality level of the units sold must be at least 7.

(b) Write down an expression for the profit.

(c) Write down an inequality that guarantees that Processes I and II operate at high enough levels that enough of Product A is produced to sell x_A, x_B, and x_C units of Products A, B, and C respectively.

(d) Using the previous answers (and possibly adding additional constraints), write down a linear programming problem that the company would solve to maximize profits subject to all of the constraints given in the problem.