Econ 172A, Fall 2008: Problem Set 2, Possible Answers

Comments: In order to get the proper sensitivity report from Excel in the Solve Parameters box select "options" and then in the Solve Options box check "assume linear model." Note: In question 3 a change in the objective function that leads to no change in the solution could lead to a change in the profits (if the coefficient is associated with a variable that is positive in the solution).

1. (a) Solution is $x_A = 3000, x_B = 2000$, $x_C = 2000$, $L_I = 3500$ and $L_{II} = 0$, with value 52000.

   (b) This is in the allowable range. Hence the solution does not change. The value goes down by 2000 (the decrease, 1, times x_B).

   (c) This is outside the allowable range (it is a decrease of 20, the solution stays the same as long as the decrease is less than 19). We know that the solution will go down by at least 19 times 2000, but to know exactly what happens, we must solve the problem again. I did this. The new solution is: $x_A = 3000, x_B = 2000$, $x_C = 0$, $L_I = 1500$ and $L_{II} = 0$, with value 14000. Note that the value went down by exactly 38000 ($= 19 \times 2000$).

   (d) I cannot solve this problem without resolving. The third constraint becomes

   $$-.5x_A + .5x_B + 1.5x_C \leq 0.$$

   I resolved the problem and got:

   $x_A = 3000, x_B = 0$, $x_C = 1000$, $L_I = 2500$ and $L_{II} = 0$, with value 25000.

   (e) This is an increase of 2000 which is in the allowable range (the allowable increase is 2500). Production plan stays the same (that is, you don't use process II and you sell some of each product). Profits increase by 4 (shadow price) times the increase. So profits increase by 8000 to 60000.

   (f) Here the change is outside of the allowable increase. You know that profits will increase by 19 for each of the first 1000 units, but you do not know what happens after that. You must resolve the problem.

   I resolved the problem and got:

   $x_A = 3000, x_B = 2000$, $x_C = 3000$, $L_I = 4500$ and $L_{II} = 0$, with value 71000.

   This is exactly what you would do with a capacity of 3000 for C. This means that the second 1000 increase in sales capacity didn't help you at all.

   (g) Same as part (d).

2. Complementary Slackness tells you that the reduced costs of x_2 and x_3 must be zero. Since $x_1 = 0$, you know that the allowable decrease is infinite and the allowable increase is the negative of the reduced costs: -3.15606936416098.

   Complementary Slackness also tells you that the first and third columns in the constraint table must be equal. Hence the missing number in LHS 1's row is 124; for LHS 2's RHS Constraint, 43; and for LHS 3's row 36.

   There is not enough information to figure out the allowable increase of x_3 and the allowable increase associated with LHS 2.

   You can figure out the final value of the problem. It is the value of the dual. You know the value of the dual variables (shadow prices): $(0.225433526, 0.757225434, 0.346820809)$. You know the value of the coefficients of the dual objective function (right-hand sides of primal): $(124, 43, 36)$. Hence

   $$0.225433526(124) + 0.757225434(43) + 0.346820809(36) = 73$$

is the value of the dual. So it is the value of the primal. But the value of the primal is also

$$x_1 + 2x_2 + 3x_3 + 4x_4$$

since the table gives you the objective function coefficients. It also tells you that $x_1 = 0, x_2 = 9, x_3 = 5$. Hence, since the value is 73, you can solve

$$18 + 15 + 4x_4 = 73$$

to conclude $x_4 = 10$.

3. (a) Profit maximizing plan is to devote 166.67 acres of the first tract to Redwoods (leave the rest open) and use all 100 acres of the second tract to camping.
   (b) $143,333.33.
   (c) Shadow price of tract 1 is zero. Hence it is not worth anything to buy more tract 1.
   (d) The shadow price is .433, which means that it is worth $433 per acre to the state to have an acre of tract 2 land. Hence it is worth buying some more at $200 per acres.
   (e) Since there is no capital and the shadow price of labor is zero, bungie jumping is always profitable on tract 1 and profitable on tract 2 if it compensates for the use of the land, that is, supplies profit of at least $433.33 per acre.
   (f) This adds .1 to the coefficient of hunting on tract 1. The change is within the allowable range so the answer (solution and value) remains unchanged.
   (g) This change subtracts .025 from the coefficient of camping in tract 2, which is in the allowable range. The solution doesn't change, but since camping is in the basis, profits go down by $25 times 100 or $2,500.
   (h) This change puts the objective function coefficient at .25, which means that it is outside of the allowable range in tract 1. All we know is that the solution will change and that profits go up.