Econ 172A, Fall 2008: Problem Set 2

Instructions: Due: November 13, 2008, in class (no late papers).

1. This is formulation problem that appeared on the midterm (with a few minor changes).

   A local company can produce three products, A, B, and C. The company can sell up to 3000 units of Product A, up to 2000 units of Product B, and up to 2000 units of Product C. Each unit of Product C uses 2 units of A and 3 units of B. Products A and B can be produced from either Process I or Process II (or combinations of these two processes). In Process I the Company can produce two units of A and three units of B for \$6. In Process II, the company can produce one unit of A and two units of B for \$5. The unit prices for the products are \$5 for A, \$4 for B, and \$25 for C. The quality levels of each product are: A, 8; B, 7; C, 6. The average quality level of the units sold must be at least 7.

   We defined variables: Let x_i be the number of units of product i sold for $i = A, B, C$. Let L_j be the level Process j is operated, for $j = I$ and II and arrived at this formulation:

$$
\begin{array}{rrrrrrrcr}
\max & 5x_A & + & 4x_B & + & 25x_C & - & 6L_I & - & 5L_{II} & & \\
\text{subject to} & x_A & & & + & 2x_C & - & 2L_I & - & L_{II} & \leq & 0 \\
& & & x_B & + & 3x_C & - & 3L_I & - & 2L_{II} & \leq & 0 \\
& -x_A & & & + & x_C & & & & & \leq & 0 \\
& x_A & & & & & & & & & \leq & 3000 \\
& & & x_B & & & & & & & \leq & 2000 \\
& & & & & x_C & & & & & \leq & 2000 \\
& & & & x, & & L & & \geq & 0 & &
\end{array}
$$

   I changed the direction of the third inequality to write the problem in standard form.

   (a) Solve the problem using Excel.

   (b) What happens to the solution and the value if the price of Product B goes down to 3?

   (c) What happens to the solution and the value if the price of Product C goes down to 5?

   (d) How does the solution of the problem change if the minimum average quality increases to 7.5?

   (e) How do profits change if the capacity to produce B increases to 4000 (from 2000)?

   (f) How do profits change if the capacity to produce C increases to 4000 (from 2000)?

   (g) Suppose that the average quality had to be 7.5 instead of 7. How would the solution change?

2. I solved a version of a linear programming problem using Excel. I attach the answer report and the sensitivity report from Excel. In these reports, I replaced several values with question marks (???). Your job is to replace these question marks with the correct information. I have not given you enough information to reconstruct the problem. You should fill in the missing values using your knowledge of Excel, duality theory, and complementary slackness. You may not have sufficient information to complete the table. If you cannot determine some of the missing numbers, then say so. If you can fill in a value, then explain what permitted you to do so. Can you determine the final value of the problem? If so, what is it?

3. The California Park Authority controls two tracts of land. Tract 1 consists of 300 acres and tract 2, 100 acres. Each acre of tract 1 can be used for redwood trees, as a wilderness preserve, or for hunting. Each acre of tract 2 can be used for redwood trees, as a wilderness preserve,

or for camping. The capital (in hundreds of dollars) and labor (in worker-days) required to maintain one acre of each tract and the profit (in thousands of dollars) per acre for each possible use of land are given in the table below. Capital of $150,000 and 200 worker-days of labor are available. I formulated the problem of maximizing profit as a linear programming problem under the assumption that the park authority can choose to leave portions of each tract unused (using no resources and earning no profit).

		Capital	Labor	Profit
Tract 1	Redwood	3	.10	.20
	Wilderness	3	.20	.04
	Hunting	4	.20	.05
Tract 2	Redwood	1	.05	.06
	Wilderness	30	5.00	.09
	Camping	10	1.01	1.10

I arrived at the following formulation:

$$
\begin{array}{rlllllll}
\max & .2x_R + & .04x_W + & .05x_H + & .06y_R + & .09y_W + & 1.1y_C & \\
\text{subject to} & 300x_R + & 300x_W + & 400x_H + & 100y_R + & 3000y_W + & 1000y_C & \le & 150000 \\
& .1x_R + & .2x_W + & .2x_H + & .05y_R + & 5y_W + & 1.01_C & \le & 200 \\
& x_R + & x_W + & x_H & & & & \le & 300 \\
& & & & y_R + & y_W + & y_C & \le & 100 \\
& x_R & x_W & x_H & y_R & y_W & y_C & \ge & 0
\end{array}
$$

In the formulation, there are six variables. x_i denotes the number of acres of the first tract allocated to use i (R stands for redwood; W for wilderness; and H for hunting). The y_i are the corresponding quantities for tract 2. The first constraint captures the limitation on capital; the second constraint states that no more than 200 worker-days of labor are available; the third and fourth constraints bound the amount of land in each tract; and the final line states that variables are non-negative.

I solved this problem using Excel. The output follows this problem. Use the output to answer the questions below. Answer the questions independently (so that a change described in one part applies only to that part).

(a) What is the profit maximizing way to allocate the tracts? (Give units.)

(b) What is the profit associated with the profit maximizing allocation? (Give units.)

(c) An economics professor owns land adjacent to tract 1. She offers to sell the land to the state for $200 per acre. The land can be used just like the land in tract one. Would the state increase its profit by buying the land? Explain.

(d) Answer the previous question assuming that the professor owned land adjacent to tract 2.

(e) A state legislator proposes to devote some parts of the land to bungie jumping. Bungie jumping requires no capital and 5 worker-days of labor per acre whether it is done in tract 1 or in tract 2. How high would the profit need to be for the state to benefit from organizing bungie jumping in tract 1? How high would the profit need to be for the state to benefit from organizing bungie jumping in tract 2?

(f) If the profit from hunting in tract 1 increased to $100 an acre, how would the solution and total profit change?

(g) If the profit from camping in tract 2 decreased by $25 an acre, how would the solution and the total profit change?

(h) Suppose rich wilderness lovers move to California and raise the profit associated with wilderness use in either tract to $250 per acre. How would the solution and total profit change?