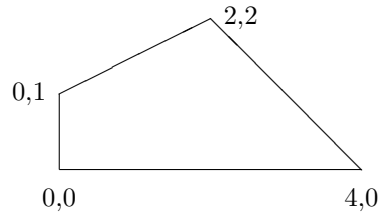


Econ 172A, Fall 2008: Problem Set 1, Possible Solutions

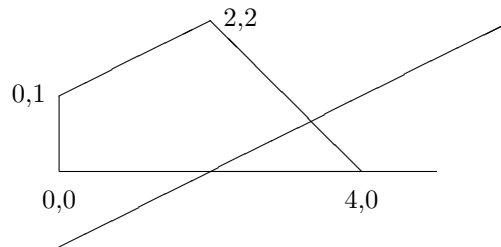
**Comments:** Ten points were possible. Students who provided no evidence of using Excel received at most six points.

1. (a) Here is a picture. (The feasible set is the figure and its interior.)



- (b) i.  $x^* = (4, 0)$ , value = 4.  
 ii.  $x^* =$  segment connecting  $(4, 0)$  to  $(2, 2)$ , value = 4.  
 iii.  $x^* = (4, 0)$ , value = 4.  
 iv.  $x^* =$  segment connecting  $(1, 2)$  to  $(2, 2)$ , value = 2.

In these answers,  $x^*$  is the name that I give to the solution. What follows is the picture with a line in which  $x_1 - 2x_2$  is constant. Shifting the line parallel, down, and to the right increases the value, hence tells you that the solution to (iii) is  $(4, 0)$ . Shifting the line parallel, up, and to the left decreases the value and (since  $\max x_0$  is the same as  $\min -x_0$ ) provides an answer to (iv)). The other two parts are similar.



- (c) The corners of the feasible set (apparent from the picture) are:  $(0, 0)$ ,  $(0, 1)$ ,  $(4, 0)$ , and  $(2, 2)$ . From (b), (i) gives an objective function with unique solution at  $(4, 0)$ . For  $(0, 0)$  one possibility is  $x_0 = -x_1 - x_2$ ; for  $(0, 1)$ , one possibility is  $x_0 = -10x_1 + x_2$ . For  $(2, 2)$   $x_0 = x_1 + 2x_2$  works. (The idea is to play around with slopes of objective functions that make different corners solutions. There are many possible solutions.)
- (d) The Excel spreadsheet contains the template for the problem. The answers are the same as the graphical answers except that Excel does not indicate multiple solutions (the particular solution the Excel finds for you will depend on how you entered the data).

- (e) This changes the problem from min to max. The solutions become: (i) value 0, segment connecting  $(0, 0)$  to  $(0, 1)$ .  
(ii) value 0, at  $(0, 0)$ .  
(iii) and (iv): solutions are part (b) (iv) and (iii) (that is, the solution to b.iii. is the solution to b.iv). Values differ by a factor of -5. That is, the value for (iii) is 15 and for (iv), -20.
- (f) This change does nothing. The answers and values stay the same. Perhaps the quickest way to see this is to graph the “new” feasible set: It is exactly the same as the old feasible set.
- (g) What happens here is that the units of  $x_2$  only are changed. The problem would be exactly the same if I created a new variable, called it  $y_2$ , set  $y_2 = 5x_2$ , and replaced  $x_2$  by  $y_2$  everywhere in the problem. Hence the values don’t change, the  $x_1$  part of the solution doesn’t change; the  $x_2$  part is multiplied by .2.
- (h) The new problem is mathematically equivalent to the original one. If you have a solution to the original problem, there are typically infinitely many solutions to the new problem: they have the same value and same  $x_1^*$  as in the original problem. If the original solution required that the second variable take on the value  $x_2^*$ , then you can solve the new problem provided that the second and third variables are non-negative and add up to  $x_2^*$ . That is, the sum of the second and third variables in the new problem.
2. Let  $p_L$  be the price of LANCE bikes and  $p_A$  be the price of ALLEN bikes. The company will make  $2000p_L + 3000p_A$  if it sells LANCES to Group I and ALLENS to Group III. This is the objective function. There are four constraints:

$$\begin{aligned} 2000 - p_L &\geq 1600 - p_A, \\ 2000 - p_L &\geq 0, \\ 3000 - p_A &\geq 2400 - p_L \end{aligned}$$

and

$$3000 - p_A \geq 0.$$

The first inequality says that Group I likes LANCES better than ALLENS. The second inequality says that Group I likes LANCES better than nothing. The third inequality says that Group III likes ALLENS better than LANCES. The fourth inequality says that Group III likes ALLENS better than nothing.

Simple algebraic manipulation lets you write these constraints so that the constants are on the right-hand side. So the problem is:

$$\begin{array}{rcllcl} \max & 2000p_L & + & 3000p_A & & \\ \text{subject to} & p_L & - & p_A & \leq & 400 \\ & p_L & & & \leq & 2000 \\ & -p_L & + & p_A & \leq & 600 \\ & & & p_A & \leq & 3000 \end{array}$$

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The solution to the problem is  $p_L = 2000$  and  $p_A = 2600$ .

3. One way to do this is to let  $x_{ij}$  be the amount of currency  $i$  exchanged for currency  $j$ . ( $x_{ii}$  is the amount you do not exchange. In this case you want to know whether it is possible, given initial holdings  $b_1, \dots, b_N$ , to find  $x_{ij}$  such that for all  $i$ ,

$$\sum_{j=1}^N x_{ij} = b_i$$

and, for all  $j$ ,

$$\sum_{i=1}^N a_{ij}x_{ij} > b_j.$$

The first set of constraints say that you use all of your money (the total amount of currency  $i$  you exchange – remembering that you can exchange currency  $i$  for itself – is equal to  $b_i$ , the total amount of currency  $i$  you start with.

The second set of constraints say that you end up with more currency  $j$  than you began with (for all  $j$ ).

If you can trade one dollar for .8 euros and 1 euro for 1 pound and 1 pound for 2 dollars, then you have an arbitrage opportunity. (Imagine that you start with 3 dollars, 2.1 euros, and 2 pounds. Exchange 3 dollars for 2.4 euros, 2.1 euros for 2.1 pounds, and 2 pounds for 4 dollars. So you end up with 1 extra dollar, .3 extra euros, and .1 extra pounds.)

In general, arbitrage is possible if and only if there is a sequence of currencies, order them  $1, 2, \dots, k$ , such that  $a_{12}a_{23} \cdots a_{(k-1)k}a_{k1} > 1$