

Econ 172A, Fall 2008: Problem Set 1

Instructions: Due: October 23, 2008, in class (no late papers). Please supply complete answers. Unless otherwise noted on homework assignments and on examinations, you are required to explain how you got your answer. Simply stating a numerical answer is insufficient. For this assignment, attach excel spreadsheets when relevant and clearly indicate the answer. This assignment asks you to solve many linear programming problems, but most are variations on the same basic problem. Set up one template for the excel computations and then make simple changes to get answers for different problems. You need not include a separate printout for every simplex computation as long as you provide a clear descriptions of how you got the answers. You are responsible for figuring out how to get Excel answers yourself (that is, I won't lecture on it). The notes on Excel should be sufficient. For this assignment there is no need to provide answer reports and sensitivity reports, but do indicate which cells on your spreadsheet have the solution. For graphs, clearly label the graph and show where the objective function is and how you identified a solution. If I ask you to solve a problem, then tell me the solution (the best x) and the value (x_0 evaluated for the best choice of x).

1. Consider the linear programming problem:

Find x to solve:

$$\begin{array}{ll} \max & x_0 \\ \text{subject to} & x_1 + x_2 \leq 4 \\ & -x_1 + 2x_2 \leq 2 \\ & x \geq 0 \end{array}$$

In this problem, x_0 is the objective function.

- (a) Graph the feasible set of the linear programming problem.
- (b) Solve the problem graphically for each of the following objective functions:
 - i. $x_0 = x_1$.
 - ii. $x_0 = x_1 + x_2$.
 - iii. $x_0 = x_1 - 2x_2$.
 - iv. $x_0 = -x_1 + 2x_2$.
- (c) Identify the corners of the feasible set. For each corner, give an example of an objective function, x_0 , such that the solution of the linear programming problem for that x_0 occurs at that corner (and only at that corner). (So you need a different x_0 for each corner.)
- (d) Solve each of the problems in the previous part using Excel. Compare your answers. Are there any differences? Explain.
- (e) Multiply each of the objective functions in part (b) by -5. Solve the new problem (any method). How do the solutions and values change?
- (f) Multiply the second constraint of the problem by 5 (so that it becomes $-5x_1 + 10x_2 \leq 10$). Resolve the problem for the objective functions in part (b) (any method). How do the solutions and values change?
- (g) Multiply the coefficient of x_2 in each constraint of the problem and in each of the objective functions in part (b) by 5. Resolve the problems. How do the solutions and values change?
- (h) Imagine that the linear programming problem changes so that there is another variable, x_3 , but the coefficient of x_3 in the objective function and all constraints are the same as the corresponding coefficient of x_2 . How does the solution of the problem change?
(Note parts (e), (f), (g), and (h) are independent. That is, for example, when you do part (f) do not multiply the objective function by 5.)

You should be able to do parts (e) through (h) with a minimum of computation. Even if you must compute, you should still be able to do these parts easily. When you get your answers, please compare them to earlier answers and comment on how they have (or have not) changed.

2. A high-end bicycle company sells two models of bicycles: road bikes (the LANCE) and triathlon bikes (the ALLEN). The company wants to sell these products to two groups of customers. Group I concentrates on just one sport and Group III concentrates on three sports. Customers in the groups differ in the value they place on bicycles. Group I customers value LANCE Bicycles at \$2,000 and ALLEN Bicycles at \$1,600. Group III customers value LANCE Bicycles at \$2,400 and ALLEN Bicycles at \$3,000. Customers will buy the bike that gives them the highest surplus, provided that the surplus is positive. (The surplus is the difference between the value and the price.) They will buy at most one bicycle. For example, if the price of a LANCE is p_L and the price of an ALLEN is p_A , then a customer in Group I will buy a LANCE if $2000 - p_L > 1600 - p_A$ and $2000 - p_L > 0$. (The first inequality says that the customer likes LANCE better than ALLEN; the second says that the customer likes LANCE better than NOTHING.)

Suppose that there are 20 Group I members and 30 Group III members. Further suppose that the company wants to maintain the distinction between the two bikes by selling LANCES to all members of Group I and ALLENS to all members of Group III. Assume that if a customer is indifferent between buying and not buying (that is, buying the better model leads to zero surplus), he will buy and if a customer is indifferent between the two models, Group I members buy LANCES and Group III members buy ALLENS. Finally assume that the company must charge the same prices to both groups (this rules out, for example charging Group I members 2,000 for LANCES, but not allowing Group III members to purchase the LANCE at any price.

Formulate the company's revenue maximization problem. Remember: A formulation should include a definition of the variables (in words), and algebraic expressions for the constraints and the objective function.

Solve the problem graphically and by using excel.

3. The rate of exchange between currencies A and B is the number of units of currency A you need to purchase a unit of currency B . For example, if a euro is worth \$1.50, then the rate of exchange between dollars and euros is 1.5 (and the rate of exchange between euros and dollars is $2/3$). An **arbitrage opportunity** exists if you can make a series of currency transactions that leave you with more money than you started with.

Suppose that there are N currencies and the rate of exchange between currency i and j is $1/a_{ij}$ (so that one unit of currency i can be turned into a_{ij} units of currency j). Assume that $a_{ij} > 0$, $a_{ii} = 1$, and $a_{ij} = 1/a_{ji}$. Write down a sequence of linear inequalities that determines whether an arbitrage opportunity exists. One possible way to do this is to assume that you have b_i units of the i th currency and you want to see if it is possible to create more than b_i units (for all i) by a series of transactions at the given rates of exchange.

Give an example of exchange rates that permit an arbitrage opportunity (you will need $N > 2$).

Give a condition on a_{ij} that rules out arbitrage opportunities.