Econ 172A, Fall 2007: Problem Set 3, Suggested Answers

1. (a) I solved the problem with the constaints that the variables must be between 0 and 1, but not necessarily integers. I obtained the value 146 (carry item 6 and .8 item 5). (I attached the spreadsheet I used for this computation.) This is the upper bound.

   Now start with the largest item (you could start with any). If we include the largest item, then we get the upper bound. If we do not take the largest item, then the solution to the resulting problem (in which you can take non-integer amounts of the first five items and cannot take the sixth item) is 134 (1/3 item 3 and items 4 and 5). No problem can be fathomed.

   We go back to the first subproblem ($x_6 = 1$). Putting $x_5 = 1$ is not feasible. Putting $x_5 = 0$ leads to a solution: $x_4 = x_6 = 1$ all others equal to zero. This yields a value 142. It is integer. Since it exceeds the upper bound of the other branch (134) it must solve the problem.

   (b) It should be clear that if $C = 10i$, then the only solution is to carry the ith item (for $i = 1, \ldots 6$).

   (c) I attach the spreadsheet.

   (d) I described the solution for 10 through 60 in part b. I used excel over and over (changing C) and filled in the results in the table below. (I did not include the spreadsheets.)

Capacity	*Weight*	1	2	3	4	5	6
70	100	1	0	0	0	0	1
80	112	0	0	1	0	0	1
90	126	0	0	1	0	0	1
100	142	0	0	0	1	0	1
110	160	0	0	0	0	1	1
120	170	1	0	0	0	1	1
130	182	0	1	0	0	1	1
140	196	0	0	1	0	1	1
150	212	0	0	0	1	1	1
160	222	1	0	0	1	1	1
170	234	0	1	0	1	1	1
180	248	1	1	0	1	1	1
190	258	1	0	1	1	1	1
200	280	0	1	1	1	1	1
210	290	1	1	1	1	1	1

   There is neither increasing nor decreasing returns. (Sometimes you gain only ten, sometimes you gain more. Notice for further increases in capacity – above 210 – you gain nothing.) Once you have room for the heaviest item, you continue to carry it. Otherwise, there is no guarantee that if the capacity increases, you will continue to carry the item. (Well, once you carry all items of at least a particular weight, then you continue to carry these.)

2. The new solution is:

3	1	0	0	2	0	0
2	5	0	0	3	0	2
4	6	0	0	5	0	0

With a value of 33. The interesting thing is that you did not need to round up all three of the original fractional values (but, of course, the value of the problem must go up).

3. First I solved the minimization problem. I subtracted constants from each row and column to obtain:

	1	2	3	4	5	6
A	6	5	0	3	83	53
B	3	0	5	7	72	52
C	0	3	5	5	71	61
D	7	9	7	0	69	19
E	20	15	25	0	0	0
F	40	0	60	40	35	15

I cannot obtain a zero-cost assignment from this table, so I need to reduce costs. I crossed out columns 2 and 4 and rows A, C, and E to get rid of 0s. The minimum uncrossed number is 3. I subtract this from everything, then add it back to all rows and columns with lines (to preserve non-negativity). I obtain:

	1	2	3	4	5	6
A	6	8	0	6	83	53
B	0	0	2	7	69	49
C	0	6	5	8	71	61
D	4	9	4	0	66	16
E	20	18	25	3	0	0
F	37	0	57	40	32	12

Still no luck. I crossed out columns 1, 2, and 4 and rows A and D, found that two was the minimum uncrossed number, and tried again:

	1	2	3	4	5	6
A	8	10	0	8	83	53
B	0	0	0	7	67	47
C	0	6	3	8	69	59
D	4	9	2	0	64	14
E	22	20	25	5	0	0
F	37	0	55	40	30	10

Still no luck. Cross out the first four columns and the fifth row and subtract ten:

	1	2	3	4	5	6
A	8	10	0	8	73	43
B	0	0	0	7	57	37
C	0	6	3	8	59	49
D	4	9	2	0	44	4
E	32	30	35	15	0	0
F	37	0	55	40	20	0

Now there is a zero-cost assignment. $A-3, B-2, C-1, D-4, E-5, F-6$. This agrees with Excel and costs 205.

To solve a maximization problem, you can convert to a minimization problem by multiplying the numbers in the original table by -1. You obtain:

	1	2	3	4	5	6
A	-8	-7	-2	-5	-100	-110
B	-6	-3	-8	-10	-90	-110
C	-4	-7	-9	-9	-90	-120
D	-8	-10	-8	-1	-85	-75
E	-90	-85	-95	-70	-85	-125
F	-80	-40	-100	-80	-90	-110

Now **add** the smallest number (that is, largest in absolute value) in each row to get a non-negative table and carry out the algorithm as before. I do not show the steps. Excel provides the answer. (You were asked to provide the details!)

4. This is a minimal spanning tree problem. Starting at (1), I connect (1) to (2). Then (1) to (3) (cheapest link from 1 and 2). Then (3) to (6). Then (6) to (5). Finally (5) to (4). The total cost is 81.

5. The table below describes how to get the shortest route to (1) from (3):

	1	2	3	4	5	6
1	18	14	0*	2	3	1**
2	18	14	0*	2**	3	1*
3	18	14	0*	2*	3**	1*
4	18	11**	0*	2*	3*	1*
5	13**	11*	0*	2*	3*	1*

The routes are:

$2 \rightarrow 1$, cost 2.

$3 \rightarrow 5 \rightarrow 2 \rightarrow 1$, cost 13.

$4 \rightarrow 6 \rightarrow 5 \rightarrow 2 \rightarrow 1$, cost 14.

$5 \rightarrow 2 \rightarrow 1$, cost 10.

$6 \rightarrow 5 \rightarrow 2 \rightarrow 1$, cost 13.