Econ 172A, Fall 2007: Problem Set 1

Instructions: Due: October 30, 2007, in class (no late papers). Please supply complete answers. Unless otherwise noted on homework assignments and on examinations, you are required to explain how you got your answer. Simply stating a numerical answer is insufficient. For this assignment, attach excel spreadsheets when relevant and indicate the answer (and the question). This assignment asks you to solve many linear programming problems, but most are variations on the same basic problem. Set up one template for the excel computations and then make simple changes to get answers for different problems. You need not include a separate printout for every simplex computation as long as you provide a clear descriptions of how you got the answers. You are responsible for figuring out how to get Excel answers yourself (that is, I won't lecture on it). The notes on Excel should be sufficient. For this assignment there is no need to provide answer reports and sensitivity reports, but do indicate which cells on your spreadsheet have the solution. For graphs, clearly label the graph and show where the objective function is and how you identified a solution. If I ask you to solve a problem, then tell me the solution (the best x) and the value (x_0 evaluated for the best choice of x).

1. Consider the linear programming problem:

   Find x to solve:

$$\begin{array}{rrcrcl} \max & x_0 & & & & \\ \text{subject to} & x_1 & + & x_2 & \leq & 3 \\ & -x_1 & + & x_2 & \leq & 1 \\ & & & x & \geq & 0 \end{array}$$

   In this problem, x_0 is the objective function.

   (a) Graph the feasible set of the linear programming problem.

   (b) Solve the problem graphically for each of the following objective functions:

      i. $x_0 = x_1$.
      ii. $x_0 = x_1 + x_2$.
      iii. $x_0 = x_1 - 2x_2$.
      iv. $x_0 = -x_1 + 2x_2$.

   (c) Identify the corners of the feasible set. For each corner, give an example of an objective function, x_0, such that the solution of the linear programming problem for that x_0 occurs at that corner (and only at that corner). (So you need a different x_0 for each corner.)

   (d) Solve each of the problems in the previous part using Excel. Compare your answers. Are there any differences? Explain.

   (e) Multiply each of the objective functions in part (b) by 5. Solve the new problem (any method). How do the solutions and values change?

   (f) Multiply the second constraint of the problem by 5 (so that it becomes $-5x_1 + 5x_2 \leq 5$). Resolve the problem for the objective functions in part (b) (any method). How do the solutions and values change?

   (g) Multiply the coefficient of x_2 in each constraint of the problem and in each of the objective functions in part (b) by 5. Resolve the problems. How do the solutions and values change? (Note parts (e), (f), and (g) are independent. That is, for example, when you do part (f) do not multiply the objective function by 5.)

   (h) Repeat parts (f) and (g), except this time multiply by -5 instead of 5.

If you think, then you should be able to do parts (f) through (h) with a minimum of computation. If you don't think, you should still be able to do these parts easily. When you get your answers, please compare them to earlier answers and comment on how they have (or have not) changed.

2. Professor Foster's family owns 125 acres of land ("Foster Farms") and has $40,000 in funds available to invest in the farm. The family works on the farm. They can supply a total of 3,500 person-hours of labor during the winter months and 4,000 person-hours during the summer. Any labor that is not used farming can be used to do odd jobs (like teaching Econ 120). These jobs pay $5.00 per hour during the winter and and $6.00 per hour during the summer. Cash income comes from three crops, soybeans, corn, and oats, and two types of livestock, cows and hens. No investments are needed for the crops. On the other hand, cows require an investment of $1,200 each, while hens cost $9 each. Each cow requires 1.5 acres of land, 100 person-hours of work during the winter and 50 hours of person-hours during the summer. Each cow produces an annual cash income of $1,000. Similarly, for each hen the farm earns $5, requires no acreage, but uses .6 person-hours during the winter and an additional .3 hours during the summer. The farm has a chicken house that has capacity of 3,000 hens and a barn that can hold up to 32 cows. The table below provides relevant information on the input requirements and profitability of the crops. The Foster family wishes to determine how much acreage should be planted in each of the crops and how many cows and hens should be kept to maximize its net cash income.

	Soybeans	Corn	Oats
Winter person-hours	20	35	10
Summer person-hours	50	75	40
Net annual cash income ($)	500	750	350

   (a) Formulate a linear program that describes the Foster family's problem.

   (b) Write the dual of the problem.

   (c) Interpret the dual.

   (d) Solve the primal and the dual using excel.

   (e) Compare the answers and confirm the conclusion of the duality theorem of linear programming and all complementary slackness conditions.

3. The manager of a processing plant needs to figure out how to schedule her workers. Each day of every working week is divided into three eight-hour shift periods (00:01-08:00, 08:01-16:00, 16:01-24:00) denoted by night, day, and late respectively. (8:00 is 8 AM, 16:00 is 4 PM, 0 and 24 are both midnight.) The plant must be manned at all times and the minimum number of workers required for each of these shifts over any working week is as below:

	Mon	Tues	Wed	Thur	Fri	Sat	Sun
Night	5	3	2	4	3	2	2
Day	7	8	9	5	7	2	5
Late	9	10	10	7	11	2	2

   The union agreement governing acceptable shifts for workers is as follows:

   (a) Each worker is assigned to work either a night shift or a day shift or a late shift and once a worker has been assigned to a shift they must remain on the same shift every day that they work.

(b) Each worker works four consecutive days during any seven day period.

Assume that the manager wishes to minimize the number of workers needed to satisfy both the scheduling constraint and the union agreement. Formulate the manager's problem as a linear programming problem. Is a linear programming formulation appropriate? That is, comment on whether there are any important assumptions made in the formulating the problem as a linear program. Use Excel to solve the problem.

Note: Your formulation must include a definition of variables (what you want to find) and clear statement – both algebraic and in words – of the objective function and all relevant constraints.

Hint: I found it convenient to use the variable x_{ij} to represent the number of workers who start working on day i at the jth shift.