

Instructions

- The examination has 5 questions. Answer them all.
- You must justify your answers to Questions 1, 2, and 5. (if you are not certain what constitutes adequate justification, please ask.) No justification is needed for Questions 3 and 4.
- You may not use books, notes, calculators or other electronic devices.
- There may be 0, 1, 2, 3, or 4 correct choices for every part of Question 3. Each part will be graded independently as explained below.
- Grading: There are 135 possible points. Question 1 is worth 21 points, 3 for each of the first two parts and 7 points for the third part and 8 points for the final part. Question 2 is worth 16 points (12 for the first part and 4 for the second part). Question 3 is worth 32 points (two points for each correct choice circled; 2 points for each incorrect choice not circled). Question 4 is worth 32 points. Each correct answer is worth four points. Question 5 is worth 34 points 4 points for each of the first five parts and 7 each for the last two parts.
- If you do not know how to interpret a question, then ask me.

	Score	Possible
I		21
II		16
III		32
IV		32
V		34
Exam Total		135

1. Consider the linear programming problem:

Find x_1 , x_2 and x_3 to solve \mathbf{P} :

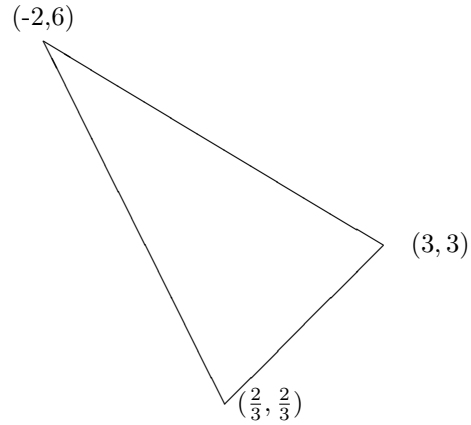
$$\begin{array}{rllll} \max & x_1 & + & x_2 & + & 4x_3 \\ \text{subject to} & x_1 & + & 2x_2 & + & x_3 & \leq & 4 \\ & 2x_1 & - & x_2 & + & x_3 & \leq & 4 \\ & x_1 & + & x_2 & & & \leq & 3 \\ & & & & & x & \geq & 0 \end{array}$$

You must provide justifications for your answers to the questions below. In particular, say what you need to do to check for feasibility and explain the basis for your inferences in part (c).

- (a) Write the dual of the problem \mathbf{P} .
- (b) Verify that $(x_1, x_2, x_3) = (2, 1, 0)$ is feasible for \mathbf{P} .
- (c) Assuming that $(2, 1, 0)$ is a solution to \mathbf{P} , use Complementary Slackness to determine a candidate solution to the dual.
- (d) Is $(2, 1, 0)$ a solution to \mathbf{P} ? Explain.

2. Convex Pizza is a producer of frozen pizza products. The company makes a net income of \$1.00 for each regular pizza and \$1.50 for each deluxe pizza produced. The firm currently has 150 pounds of dough mix and 50 pounds of topping mix. Each regular pizza uses 1 pound of dough mix and 4 ounces (16 ounces = 1 pound) of topping mix. Each deluxe pizza uses 1 pound of dough mix and 8 ounces of topping mix. Based on the past demand per week, Convex can sell at least 50 regular pizzas and at least 25 deluxe pizzas. The problem is to determine the number of regular and deluxe pizzas the company should make to maximize net income. Formulate this problem as an LP problem. Your formulation should include a definition of the variables (in words).

Suppose now that Convex can sell any topping mix for 50 cents per pound. How does the formulation change (provide the new formulation)?



3. The following questions relate to the triangle above (and its interior), which I call S . You should think of S as the feasible set of a linear programming problem.

Remember, any number of choices (from zero to four) can be correct. As described on the first page, you receive credit for each correct choice you select and for each incorrect choice you do not select.

- (a) The triangle above (and its interior) is described by which of the following sets of linear inequalities.

i.
$$\begin{aligned} 3x_1 + 5x_2 &\leq 24 \\ x_1 - x_2 &\leq 0 \\ 2x_1 + x_2 &\geq 2 \\ x_2 &\geq 0 \end{aligned}$$

ii.
$$\begin{aligned} 3x_1 + 5x_2 &\leq 24 \\ x_1 - x_2 &\geq 0 \\ 2x_1 + x_2 &\geq 2 \end{aligned}$$

iii.
$$\begin{aligned} 3x_1 + 5x_2 &\leq 24 \\ x_1 - x_2 &\geq 0 \\ 2x_1 + x_2 &\geq 2 \\ x_1 &\geq 0 \end{aligned}$$

iv.
$$\begin{aligned} 3x_1 + 5x_2 &\geq 24 \\ x_1 - x_2 &\leq 0 \\ 2x_1 + x_2 &\geq 2 \end{aligned}$$

Consider the problem of finding x_1 and x_2 to solve the linear programming problem $\max x_0$ subject to $(x_1, x_2) \in S$, where x_0 is a linear function of x_1 and x_2 that is not constant (so $x_0 = Ax_1 + Bx_2$ and at least one of A and B is not zero). Call this problem **P**.

- (b) For which of the following specifications of x_0 does the problem **P** have a **unique** solution?
- i. $x_0 = x_1$
 - ii. $x_0 = x_1 - x_2$
 - iii. $x_0 = 5x_1 - 3x_2$
 - iv. $x_0 = -2x_1 - x_2$
- (c) Suppose that x_0 is a function with the property that $(3, 3)$ solves **P**. For which of the following functions y_0 **must** it be the case that $(3, 3)$ solves $\max y_0$ subject to $(x_1, x_2) \in S$?
- i. $y_0 = x_0 + x_1$
 - ii. $y_0 = x_0 + x_2$
 - iii. $y_0 = 5x_0$
 - iv. $y_0 = -x_0$
- (d) For which of the following pairs of points is it possible to find a non-constant x_0 such that both points solve **P**?
- i. $(2, 2)$ and $(\frac{2}{3}, \frac{2}{3})$
 - ii. $(4, 4)$ and $(3, 3)$
 - iii. $(-2, 6)$ and $(2, 2)$
 - iv. $(\frac{2}{3}, \frac{2}{3})$ and $(-2, 6)$

4. For each of the statements below indicate whether the statement is always **TRUE**, by writing “TRUE” otherwise write “FALSE.” No justification is required.

- (a) If a linear programming problem has more variables than constraints it is feasible.
- (b) If a linear programming problem is unbounded, then it will continue to be unbounded if the objective function changes.

The next six parts refer to the linear programming problem (P) written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and its dual (D):

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0$$

- (c) If (P) has a unique solution, then its dual has a solution.
- (d) If x^* is a solution to (P), then x^* will be a solution to

$$\max rc \cdot x \text{ subject to } Ax \leq b, x \geq 0 \text{ for any } r.$$

- (e) If (P) has a solution and $\bar{b} \geq b$, then

$$\max c \cdot x \text{ subject to } Ax \leq \bar{b}, x \geq 0$$

has a solution.

- (f) If (D) has a solution, then the problem:

$$\max c \cdot x \text{ subject to } Ax \leq b/4, x \geq 0$$

is feasible.

- (g) If (P) has a solution, x^* , then there exists a solution to (D), y^* , such that $b \cdot y^* = y^* Ax^*$.
- (h) If (P) has a solution and $c' \geq c$, then

$$\max c' \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

has a solution.

5. A local restaurant makes three different kinds of burger. A classic cheeseburger uses one-quarter pound of ground beef and a slice of cheese. A turkey burger uses one quarter pound of ground turkey. A double cheese burger uses one-half of a pound of ground beef and one slice of cheese. In addition, each type of burger requires a bun. The restaurant can sell a classic cheese burger for \$4.00, a turkey burger for \$5.50, and a double (cheese) burger for \$8.00. Each day the restaurant has available 800 pounds of ground beef, 500 pounds of ground turkey, 2500 buns, and 2000 slices of cheese. The manager insists that the restaurant make at least 100 double cheese burgers every day. The restaurant wishes to know how many burgers of each type to produce in order to maximize profits subject to the constraints above. In order to formulate the problem, I defined the variables:

x_1 = number of cheese burgers produced.

x_2 = number of turkey burgers produced.

x_3 = number of double burgers produced.

The problem is then: find values for x_1 , x_2 , and x_3 to solve:

$$\begin{array}{rcllcl}
 \max & 4x_1 & + & 5.5x_2 & + & 8x_3 & & \\
 \text{subject to} & & & \frac{x_2}{4} & & & \leq & 500 \\
 & x_1 & & & + & x_3 & \leq & 2000 \\
 & \frac{x_1}{4} & & & + & \frac{x_3}{2} & \leq & 800 \\
 & x_1 & + & x_2 & + & x_3 & \leq & 2500 \\
 & & & & & x_3 & \geq & 100
 \end{array}$$

$$x \geq 0.$$

I solved this problem using Excel. The output follows this problem. Use the output to answer the questions on the next page. Answer the questions independently (so that a change described in one part applies only to that part). You must justify your answers by providing brief (but complete) descriptions of how you arrived at them.

- (a) What is the restaurant's profit maximizing output? How much does it earn?
- (b) What is the most that the restaurant would be willing to pay for an additional pound of ground beef?
- (c) What is the most that the restaurant would pay for another slice of cheese?
- (d) Would the restaurant produce more double burgers if it could sell them for \$9 each?
- (e) How much would the profits of the restaurant change if received 100 extra buns?
- (f) An employee suggests making Hawaiian burgers by replacing the cheese on a double burger with a slice of pineapple. Pineapples cost 50 cents per slice. Would the restaurant want to put Hawaiian burgers on the menu if it could sell them for \$7.50 each?
- (g) The restaurant determines that it could sell a low carb cheeseburger that contains two slices of cheese, one-third of a pound of ground beef, but does not use a bun. What is the lowest price the restaurant could charge for this kind of burger (without lowering its profits)?