Econ 172A, Fall 2004: Midterm II-A, Suggested Answers

Comments: There were 100 points possible. Scores ranged from 19 to 100 with a median of 68. As promised in the course outline, here are the percentile breakdowns for letter grades (I do not promise to follow these guidelines). Relatively easy grading scale:

Grade	Points on Midterm 2	Points on Midterm 1 + 2
A	85	166
B	63	137
C	27	84

and the harder scale

Grade	Points on Midterm 2	Points on Midterm 1 + 2
A	87	169
B	72	148
C	46	114

Here is how to interpret these tables. The second column gives you the minimum number of points on Midterm 2 you must have in order to get the indicated grade using each scale. So, according to the easy scale, 63 points on the second midterm is the lowest B. The third column gives the information cumulatively (first two midterms). I think that the harder scale is more accurate (people at the bottom of the class tend to drop, raising the threshold for higher grades). I repeat, I provide this information for guidance only. I do not promise to follow either scale when assigning final grades. (I am still hoping that everyone earns the highest grade.)

If you have questions or complaints about the way your paper was graded, please follow the directions for regrading that I have posted on the webpage.

Comments: Here are notes on how points were assigned.

1. Two points for (a) and four four each part of (b); ten points total; no partial credit.
2. (a) 4 points; one point deduction for slips (like no nonnegativity constraints)
   (b) 4 points: I hope that this is a gift. Check all constraints (including non negativity).
   (c) Solution must note (i) dual constraints binding (and why) (3 points); (ii) $y_2 = 0$ (and why) (3 points); (iii) solve the right system of equations (3 points) (9 points total).
   (d) The best way to answer the question is by checking dual feasibility of what they came up with in (b) and giving a reason (CS; duality theorem) (4 points). On this question, students should not be counted off two times for the same mistake. For example, if they did part (c) incorrectly and arrived at the wrong answer, they can get full credit in (d) for correctly completing the evaluation.
3. (a) 4 points (as in 2a).
   (b) 19 points total Definition of variables: 2 point for correct units; 3 point for "they are prices;" 3 more points for correct description of what the prices mean.

   5 points each for a discussion of objective function; 6 points for correctly interpreting the constraints, including discussing the value of the contracted deliveries. 5 points without discussing the contracted deliveries. 2 points for linking the constraints to something having to do with profit. 1 point for linking the constraints to something having to do with red, white, and blue wines separately.

(c) 10 points total. Full credit for exhibiting a solution or for showing both primal and dual are feasible. 5 points for accurately describing any technique that will allow them to answer the question and 2 more points for describing correctly how they would execute the technique.

4. Give 4 points for part (a); 5 points for b-e; and 6 points for f and g for a total of 36 points. Deduct for incorrect or absent reason. (The reason can be brief: "this change is within the allowable range" or "because the constraint is not binding.")

Suggested Solutions There were two forms that differed in small ways. Most of my comments refer to Form A, with additional remarks indicating the difference, if any, between Form A and Form B.

1. (a) The dual of **P** has three variables, one for each primal constraint. FORM B: two variables.
   (b) i. The first dual constraint must be binding.
   ii. None. CS says that if a primal constraint is not binding the associated dual variable must be zero, but it never requires a dual variable to be positive.

2. Consider the linear programming problem:

   Find x_1 and x_2 to solve **P**:

$$\begin{array}{lrcrcl} \max & x_1 & + & x_2 & & \\ \text{subject to} & x_1 & + & 2x_2 & \leq & 4 \\ & 2x_1 & - & x_2 & \leq & 4 \\ & x_1 & + & x_2 & \leq & 3 \\ & & & x & \geq & 0 \end{array}$$

(a)
$$\begin{array}{lrcrcrcl} \min & 4y_1 & + & 4y_2 & + & 3y_3 & & \\ \text{subject to} & y_1 & + & 2y_2 & + & y_3 & \geq & 1 \\ & 2y_1 & - & y_2 & + & y_3 & \geq & 1 \\ & & & & & y & \geq & 0 \end{array}$$

Form B:

$$\begin{array}{lrcrcrcl} \min & 4y_1 & + & 4y_2 & + & 3y_3 & & \\ \text{subject to} & 2y_1 & + & 4y_2 & + & 2y_3 & \geq & 1 \\ & 2y_1 & - & y_2 & + & y_3 & \geq & 1 \\ & & & & & y & \geq & 0 \end{array}$$

(b) Just plug the values into the constraints to confirm that the first and third constraints hold as equations and the second one holds as a strict inequality.

(c) Since $x_1^*, x_2^* > 0$, both dual constraints bind. Since the second primal constraint is not binding, $y_2^* = 0$. Now solve: $y_1^* + y_3^* = 1$ and $2y_1^* + y_3^* = 1$ to get $y_1^* = 0$ and $y_3^* = 1$. So the candidate solution to the dual is $y^* = (0, 0, 1)$. FORM B: The right-hand side of the first dual constraint should be 2 (not 1). solve: $2y_1^* + 2y_3^* = 2$ and $2y_1^* + y_3^* = 1$ to get $y_1^* = 0$ and $y_3^* = 1$. So the candidate solution to the dual is $y^* = (0, 0, 1)$.

(d) $(2, 1)$ a solution to **P** because the associated candidate solution to the dual is feasible for the dual (satisfies all the constraints). To see this, we need to check that y^* is non-negative, because by construction it satisfies both dual constraints. Non-negativity is obvious. FORM B: Same.

3. (a)

$$\begin{array}{lrcrcrcrcl}
\text{min} & 220y_1 & + & 480y_2 & + & 3000y_3 & - & 60y_4 & & \\
\text{subject to} & y_1 & + & 6y_2 & + & 1.5y_3 & - & y_4 & \geq & 5 \\
 & 2y_1 & + & 8y_2 & + & 1.5y_3 & & & \geq & 8 \\
 & 3y_1 & + & 8y_2 & + & 2y_3 & & & \geq & 4 \\
 & & & & & & & y & \geq & 0
\end{array}$$

(b) Interpreting the dual as a buy-out problem, y_i are prices. y_1 is the price offered for an hour of processing time (the units are dollars per hour); y_2 is the price of an hour of bottling time (again, the units are dollars per hour); y_3 is the price of space in the warehouse (dollars per cubic feet); and y_4 is the price you must pay to the buy out artist to get her to take over the contractual obligation (dollars per bottle of red wine).

The buy out artist offers to buy out all of Pallo's assets. She wants to do this at minimum cost. The first three terms in the objective function are straightforward. The fourth term enters with a negative sign because the contractual requirement to supply red wine is costly to Pallo. Hence the buy out artist receives a payment to take it over. The constraints guarantee that the buy out provides at least as much money to the wine maker as making wine. In each case, the constraints say that the value of the raw materials must be at least as large as the sale price of the final product. The red wine constraint is the most complicated. The wine company has the choice of selling red wine for \$5 or selling the ingredients are earning: y_1 (from "selling" processing time); $6y_2$ (from selling six hours of bottling time); $1.5y_3$ (from warehouse space). In addition, the wine company will have to pay the buy out artist y_4 in order to discharge the obligation to deliver red wine.

(c) The primal is feasible. You can set $x_R = 60$ and $x_W = x_B = 0$. This satisfies the constraints. (You set $x_R = 60$ because the last constraint requires that x_R is at least 60.) The dual is feasible. For example, set $y_2 = y_3 = y_4 = 0$ and $y_1 = 5$ (big enough to satisfy the constraints.) Since both Primal and Dual are feasible, by the duality theorem, both must have solutions.

4. Form B differs from A only because oatrye uses .5 pound of oatmeal instead of .25 pounds.

(a) The bakery earns 310 by producing 5 loaves of whole wheat bread and 120 loaves of oatrye bread.

FORM B: It earns 280 by producing 65 whole wheat loaves and 60 oatrye loaves.

(b) Nothing (the white flour constraint is not binding). FORM B: same.

(c) \$2 (the dual price of constraint that describes the oatmeal constraint). FORM B: \$1.

(d) The allowable increase on the coefficient of oatrye in the objective function is infinity. Hence doubling its price would not change the solution (it would increase profits). FORM B: SAME

(e) Constraint 7's right hand side goes down by 5. This is in the allowable range. Hence profits decrease by the value of the dual variable for that constraint, 2, times 5. Profits decrease by 10. FORM B: SAME

(f) The baker was not producing white bread before. It is as if he or she is given an opportunity to produce a new product using .75 pounds of white flour, 2 ounces of yeast, and .5 "spaces" of the oven. The value of these ingredients (using the dual prices) is 1, which is less than the price of white bread (1.5). Hence it pays to produce white bread under these circumstances. FORM B: SAME

(g) The value of the ingredients is 2.50 (\$2 for the oven space and .25(\$2) for the oatmeal). The other ingredients are in excess supply and therefore are available without cost. Hence if the baker can sell the new bread for more than \$2.50 it will be profitable to do so. FORM

B: Same, except that not the dual variable for oatmeal is \$1, so the value of ingredients is only \$2.25.