

Econ 172A, Fall 2002: Midterm Examination IIa

Instructions.

1. The examination has three questions. Answer them all.
2. If you do not know how to interpret a question, then ask me.
3. You must justify your answer to the Questions 1 and 2. On this question it is important that you carefully define your variables, clearly identify the objective, and explain for the constraints correspond to the problem statement. No justification is necessary for Question 3.
4. The table below indicates how points will be allocated on the exam.
5. Work alone. You may not use notes or books.
6. You have until 9:20 to finish the exam.

	Score	Possible
I		40
II		40
III		20
Total		100

1. Ron En has a new company that sells two kinds of cattle feed. Both types are mixtures of wheat and alfalfa. Feed X must contain at least 80% wheat and Feed Y must contain at least 60% alfalfa. Feed X sells for \$1.50 per pound and Feed Y sells for \$1.30 per pound. En can purchase up to 1000 pounds of wheat at 50 cents per pound and up to 800 pounds of alfalfa at 40 cents per pound. Demand for each type of feed is unlimited. Formulate a linear programming problem that determines how to maximize the company's profit. Your formulation must contain: a definition, in words, of the variables (specify the units of the variables); the objective function; and all relevant constraints.

2. A computer firm manufactures two models of microcomputers, the Lemon and the Banana. The firm employs fifty workers; each works 160 hours per month. No overtime is available (but workers need not work full time). It requires 18 hours of labor to assemble a Banana and 25 hours to assemble a Lemon. The company wants to produce at least 100 Bananas, but no more than 200 Lemons, during the next month. Bananas generate \$1200 profit per unit, and Lemons yield \$1800 each. I formulated this problem below. My variables are x_L , which represents the number of Lemons produced, and x_B , which represents the number of Bananas produced. The problem is to find $x = (x_L, x_B)$ to solve:

$$\begin{array}{rcll}
 \max & 1800x_L + 1200x_B & & \\
 \text{subject to} & 25x_L + 18x_B & \leq & 8000 \\
 & & x_B & \geq 100 \\
 & x_L & \leq & 200 \\
 & & x & \geq 0
 \end{array}$$

- (a) Write the dual of this problem.
- (b) The boss proposes that you produce 200 Lemons and $166\frac{2}{3}$ Bananas ($x_L = 200$ and $x_B = 166\frac{2}{3}$). (The formulation assumes that it is possible to produce and sell fractional computers.)
- Is this production plan feasible?
 - How much revenue does this production plan generate?
 - Does the production plan solve the optimization problem? If so, exhibit a solution to the dual. I recommend that you use complementary slackness to answer this question. You may use another method, but in any case you must provide a complete explanation for your answer.
- (c) Provide an interpretation of the dual problem. Your interpretation should contain a definition in words of each of the dual variables and an explanation of the dual constraints in a way that relates to the original problem. You **must** specify the units of each of the dual variables and each of the dual constraints.

3. Suppose that you solve a pair of linear programming problems, a primal, (P):

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and its dual, (D):

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0.$$

Assume that both problems have at least three constraints (in addition to the non-negativity constraints), x^* is the solution to (P), and y^* is the solution to (D).

Decide whether each statement below is always true, sometimes true, or never true (under the stated conditions of the problem). That is, for each statement below, write “always,” if the statement is true; “sometimes,” if there exists problems (P) and (D) above, with solutions x^* and y^* , such that the statement is true **AND** there exist (different) problems and solutions such that the statement is false; and write “never” if the statement is never true.

(a) Assume that $x_1^* > 0$ and the first constraint in the primal is not binding.

- i. The first constraint in the dual is binding.
- ii. The second constraint in the primal is binding.
- iii. $y_1^* > 0$.
- iv. The value of the primal is greater than or equal to $c_1 x_1^*$.
- v. $y^* Ax^* = b \cdot y^*$.

(b) Assume that no dual constraint binds.

- i. At least one primal constraint binds.
- ii. $x^* = 0$.
- iii. $b \geq 0$.
- iv. Increasing c would not change the value of the primal.
- v. Decreasing c would not change the value of the dual.