Econ 172A, Fall 2004: Midterm I
Possible Answers

Comments The exam had 100 points. The high was 98. The low was 40. In the outline I promised to give approximate grading information. I reported that over the past few years my "easiest" distribution gave 16% A; 41% B; 41% C; and 2% lower and my "hardest" distribution gave 12% A; 28% B; 44% C; and 16% lower. On this exam, these percentages translated to points as follows. For the more generous scale: A at least 86; B at least 79; C at least 64. For the less generous scale: A at least 84; B at least 74; C at least 50. Use these numbers as guides. When I assign final grades I will first add up your points (weighted according to the percentages in the course outline). I will not refer to the rough letter grades for the first exam.

Note: The order of the detailed answers below corresponds to form A. The choices on Form B were in slightly different order, but the actual questions were the same. Before you compare your answers to the solutions, make sure you check to see that you are using the correct form.

1. (a) (ii) and (iv)

    Choices (i) is out because it has a negative value; choice (iii) is out because the value of the objective function goes down; choice (ii) is fine (it comes from pivoting on x_2 column and row (3)); choice (iv) is fine (it comes from pivoting on x_3 column and row (3)).

    (b) None. ((ii) and (iii) both have negative numbers in row 0).

    (c) All but (iv). (You can plug them into the equations represented by the original array.)

    On Form B: (a) (ii) and (iii); (b) none; (c) all but (iv).

2. (a) (i) and (iv)

    (b) (i), (ii)

    You can rule out (iii), because everything connecting $(-2, 6)$ and $(\frac{2}{3}, \frac{2}{3})$ is a solution; you can rule out (iv) because everything between $(\frac{10}{3}, \frac{10}{3})$ and $(\frac{2}{3}, \frac{2}{3})$ is a solution.

    (c) (i) and (iii). In the case of (i), you make the objective function steeper, which makes $(\frac{10}{3}, \frac{10}{3})$ even more attractive. For (iii), multiplying the objective function by a positive constant never changes the solution. For (ii) the change flattens the slope, whether the change is sufficient to shift the solution to $(-2, 6)$ depends on x_0. If $x_0 = x_1$ then the change doesn't influence the solution; if $x_0 = .1x_1$, then the change does move the solution.

    (d) (i), (ii) and (iv): You can achieve any points on an edge of the feasible set as solutions. It is clear that the first and last are on the same edge. It is not hard to check that $(5, 5)$ is on the segment connecting $(-2, 6)$ and $(\frac{10}{3}, \frac{10}{3})$.

    On Form B: (a) (iii) and (iv); (b) (i) and (ii); (c) (ii) and (iii); (d) (i), (ii), and (iii).

3. (a) If a linear programming problem has more constraints than variables it is not feasible.
    False. Not close.

    (b) If a linear programming problem is not feasible, then it will continue to be infeasible if the objective function changes.
    True. Feasibility is a property of constraints and not objective function.
    The next six parts refer to the linear programming problem (P) written in the form:

    $$\max c \cdot x \text{ subject to } Ax \le b, x \ge 0$$

    and its dual (D):

    $$\min b \cdot y \text{ subject to } yA \ge c, y \ge 0$$

(c) If (P) has a unique solution, then its dual has a unique solution.

False. Not close.

(d) If x^* is a solution to (P), then rx^* will be a solution to

$$\max c \cdot x \text{ subject to } Ax \leq rb, x \geq 0 \text{ for any } r > 0.$$

NOTE: On the exam I left out "max" in the objective function. Consequently the problem made no sense. True. rx^* is feasible for the new problem. It must solve the new problem. Otherwise, there would be a feasible y^* that leads to a higher value of the objective function for the new problem. This implies that $\frac{y^*}{r}$ would be better for the original, which is a contradiction.

(e) If (P) has a solution, then

$$\max c \cdot x \text{ subject to } Ax \leq \bar{b}, x \geq 0$$

has a solution ($\bar{b}$ may be different from b).

No. The second problem may be infeasible.

(f) If (P) has a solution, then the dual of the problem:

$$\max 4c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

is feasible.

Yes. A solution to (P) will also solve the new problem, so its dual has a solution (and so is feasible).

(g) If (P) has a solution, x^*, then there exists a solution to (D), y^*, such that $y^* = x^*$.

No, this is nonsense. The duality theorem implies that $b \cdot y^* = c \cdot x^*$, but in general y^* will have a different number of components than x^*.

(h) If (P) has a solution and $c' \leq c$, then

$$\max c' \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

has a solution.

This is true. The new problem is still feasible. The dual to the new problem must be feasible because its feasible set is at least as large as the feasible set of (D). Specifically, if y^* is the solution to the original dual (which we know exists because (P) has a solution), then $y^*A \geq c, y^* \geq 0$. Since $c \geq c'$, it must be that $y^*A \geq c'$, so the new dual is feasible. Since the new primal and its dual are both feasible, they must both have a solution.

On Form B: (a), (f), (g) TRUE; (d) TRUE or FALSE; the others – FALSE.