Econ 172A, Fall 2004: Final Examination (I)

Instructions.

1. The examination has five questions. Answer them all.
2. If you do not know how to interpret a question, then ask me.
3. You must justify your answers to each question.
4. The table below indicates how points will be allocated on the exam.
5. Work alone. You may not use notes, books, or calculators.
6. You have three hours.
7. If you sign the Buckley waiver attached to the exam, then you will be able to pick up your exam in a public area in Sequoyah 245 when the tests are graded (on or before December 15, 2004). If you do not sign the Buckley waiver, then you will be able to pick up your exam from the department undergraduate coordinator beginning January 18, 2005. I will accept requests to reconsider grades only from students who sign the Buckley waiver **and** submit a request in writing after examining (**but not removing**) their examination from Sequoyah Hall. (I try to be especially careful when evaluating borderline cases and have changed about five final grades in the last 26 years.)

	Score	Possible
I		50
II		40
III		40
IV		60
V		60
Exam Total		250
Course Total		500
Grade in Course		

1. My son Ben is starting a rock band, Lit Fuse. Ben will do the vocals and play lead guitar. Five of his friends will be in the band. They have different talents. Ben evaluated the relative abilities of his friends and wants to come up with an assignment that maximizes the total quality of the band. The table below gives the benefit of assigning a particular boy to a particular instrument. (So, for example, if Adam plays Bass, Alex plays Drums, Chase plays guitar, Isaac plays Keyboard, and Ryan is the Roadie, then the quality if the band is: $10 + 13 + 18 + 12 + 10 = 63$.)

	Bass	Drum	Guitar	Keyboard	Roadie
Adam	10	12	6	8	5
Alex	17	13	10	16	2
Chase	15	5	18	11	6
Isaac	14	6	16	12	4
Ryan	14	6	16	12	10

   (a) Assume that each boy plays exactly one role (instrument or "roadie") and each role is assigned to exactly one boy. Find an assignment of boys to instruments that maximizes the total expected profit. Your answer should describe which boy should be assigned to which role and the associated total benefit. You **must** explain how you arrived at the answer and why it solves the problem. (Properly using the algorithm presented in class, with short explanations of the steps, is sufficient. If you do not use the algorithm, you must provided complete and detailed arguments to justify your answer.) Please note that the objective is to maximize total benefit.

   (b) Suppose that Chase gets a drum set for Christmas and therefore becomes the band's drummer. What is the optimal assignment of roles for the rest of the band? (Again, your answer should describe which boy should be assigned to which role and the associated total benefit. You **must** explain how you arrived at the answer and why it solves the problem.)

2. A fertilizer company has decided to manufacture a large supply of various plant foods to be sold during the upcoming planting season. The company can invest up to $25,000 in the three basic ingredients: nitrates, which cost $800 per ton; phosphates, which cost $400 per ton; and potash, which costs $1000 per ton. Three standard grades of plant food will be produced from these ingredients: regular, in which nitrates, phosphates, and potash are combined, respectively, in a 3:6:1 ratio by weight; extra is a 4:4:1 mixture; super is a 6:4:3 mixture. Regular can be sold for $750 per ton. Extra can be sold for $800 per ton. Super can be sold for $900 per ton. The company's objective is to maximize profits (total sales minus total expenditures for ingredients). Its production capacity permits it to manufacture no more than 40 tons of plant food overall.

   (a) Formulate an LP that would determine how much of each ingredient it should buy and how much of each grade of plant food it should produce.

   (b) Repeat part (a) subject to the additional condition that the firm can somehow earn an immediate 10% on all capital **not** invested in nitrates, phosphates, and potash. Hence, the firm can earn $1 on each $10 not spent on the three ingredients.

   Both of your answers must include a definition of the variables, in words. The definition must include appropriate units for the variables. You must also specify the objective function (and explain why the function you write down is appropriate) and all constraints (along with a description of how they correspond to the problem description).

3. For what values of A is $(x_1, x_2, x_3, x_4) = (8, 0, 0, 3)$ a solution to the linear programming problem:

$$\begin{array}{rrcrcrcrcl} \max & Ax_1 & - & 4x_2 & - & 6x_3 & + & 5x_4 & & \\ \text{subject to} & x_1 & + & 4x_2 & + & 8x_3 & - & 2x_4 & \leq & 2 \\ & -x_1 & + & 2x_2 & + & 4x_3 & + & 3x_4 & \leq & 1 \\ & & & & & & & x & \geq & 0 \end{array}$$

You may use any method to answer this question, but you must explain the method that you use and why it works.

4. Jack and Jill live together. Jack works at home and likes to smoke. Jill, who must spend much of the day away from home, hates it when Jack smokes. Imagine that each day Jack can do one of four things: he can abstain from smoking; he can smoke at 1 PM; he can smoke at 2 PM; or he can smoke at 3 PM (he never smokes more than once in a day). Jill can check up on Jack exactly once (at 1, 2, or 3). When Jill checks, she can detect signs of smoking in the previous hour (so if Jill checks at 2 PM, she'll know if Jack smoked at 1 or at 2). Assume that Jack and Jill choose their strategies simultaneously. If Jack does not smoke, he receives payoff 0. If he smokes and Jill does not find out, he receives payoff 1. If he smokes and Jill catches him, he receives payoff -1. The game is zero sum.

   Answer the questions on the next page. Your answers should provide sufficient justification that it is clear you understand the relevant concepts (dominance, value, and so on).

(a) Write down a payoff matrix for this game. (Label the strategies and explain what they represent.)

(b) Does Jack have any dominated strategies? If so, identify the dominated strategies and explain why they are dominated.

(c) Does Jill have any dominated strategies? If so, identify the dominated strategies and explain why they are dominated.

(d) Find the pure-strategy security levels of both players.

(e) Does the game have an equilibrium in pure strategies? If so, find it.

(f) Now assume Jill's checking strategy is constrained in the following way: the probability that she checks at 1 PM must be equal to the probability that she checks at 2 PM. Write down a payoff matrix for this game. (To do this, assume that Jill has two pure strategies: "check at 3 PM" and "check at 1 PM with probability $\frac{1}{2}$ and check at 2 PM with probability $\frac{1}{2}$").

(g) Find the equilibrium strategies and the value of the game you wrote in part (f).

(h) What does your answer to part (g) tell you about the equilibrium strategies and value of the original game? (Answer as completely as possible).

5. A drug company can sell three different products, Drug A, Drug B, and Drug C at the (respective) price of \$8, \$70, and \$100 per unit. Producing one unit of A requires one hour of labor. Producing one unit of B requires two hours of labor and two units of A. Producing a unit of C requires three hours of labor and one unit of B. Any product A used to produce B cannot be sold, and any product B used to produce C cannot be sold. There are 40 hours of labor available. What follows is a linear programming formulation of the problem that the company solves in order to maximize revenue.

   Define the following variables: S_i = amount of drug i sold, for $i = A, B, C$. T_i = total amount of drug i produced, for $i = A, B, C$.

   The firm's optimization problem is then:

$$\begin{array}{lrcrcrcrcrcrcr} \max & 8S_A & + & 70S_B & + & 100S_C & & & & & & & & \\ \text{subject to} & S_A & & & & & - & T_A & + & 2T_B & & & \leq & 0 \\ & & & S_B & & & & & - & T_B & + & T_C & \leq & 0 \\ & & & & & S_C & & & & & - & T_C & \leq & 0 \\ & & & & & & & T_A & + & 2T_B & + & 3T_C & \leq & 40 \end{array}$$

   and all variables nonnegative.

   I attached Excel's solution to the exam (following the questions). Answer as many of the questions below using the output. If there is some question that you cannot answer using the output that I have provided, explain why you cannot answer it (and say as much as you can). Answer each question independently of the other parts. Justify your answers.

(a) Interpret the constraint: $S_A - T_A + 2T_B \leq 0$.

(b) What is the solution to the company's problem? How much profit does it make?

(c) If the unit price for A doubles, does the solution change? If so, how?

(d) What would happen to the solution if the price of B fell to 55?

(e) How much would the company be willing to pay for an extra hour of labor?

(f) How would the profit and maximum revenue change if the seller was taxed $ 1 for every unit of drug A produced?

(g) Would it be worthwhile for the firm to produce a drug that requires three units of labor and one unit of drug B and sells for $150 per unit.

(h) Interpret the shadow price associated with the constraint: $S_C - T_C \leq 0$.