Econ 172A, W2002: Midterm Examination I, Answers

Comments There are two forms. I give the answers first for the form called Examination I on the front page. The answers prefaced by "on the other form" are for Exam Ia.

There were 140 points on the exam. The first two questions were worth 30 each, the remaining questions 20 each. The scores ranged from 74 to 132, the mean was 104, and the median was 106. Answers were good on Questions 1 and 3 and less good on Questions 2 and 4. On Question 2 you needed to realize that neither variable was explicitly constrained to be non-negative, so that something needed to be done (like the $x = u - v$ trick) for both x_1 and x_2). If you failed to do this correctly in part (a), you were not penalized again in subsequent parts of the question. On the second question it was important to write the primal in the proper form in order to get the answer to the dual correct. Answers to Question 4 were not good, but it was hard to judge what you were thinking. On Question 5, the line segment is a possible feasible set (for example $x_1 + x_2 \leq 1, x \geq 0$), but the set that looked like everything but the positive orthant was not.

If you have questions about your exam, I encourage you to speak with me. If you wish to have portions of your exam regraded, you **must** follow the rules below.

1. Please read the posted answers to the exam carefully.

2. You have two weeks from the date of the quiz to request a regrade. (That means that you have until February 21 to request a regrade of Midterm 1.) After this date, do not bother.

3. If you have questions about how your exam was graded and you think that your score should be adjusted, then submit your question to me **in writing**.

   (a) Use a separate sheet of paper. (Do **not** write anything on your exam paper if you want your score adjusted.)

   (b) On the separate sheet, explain clearly what you do not understand about the grading of the exam.

4. The grader and I will both review the grading of the entire exam, with an emphasis on your question. We will respond to your question.

5. If you disagree with or do not understand our comments, then speak to me about it in office hours.

1. Consider the linear programming problem:

$$\max x_0$$

$$\begin{array}{rrcrcr} \text{subject to} & 5x_1 & + & 3x_2 & \leq & 30 \\ & 5x_1 & - & x_2 & \geq & 10 \\ & & & x_2 & \geq & -7 \end{array}$$

   (a) Graphically represent the feasible set of this problem. The feasible set is a triangle with vertices $(3,5)$, $(.6,-7)$, and $(10.2,-7)$.

   (b) Graphically solve the problem for the following values of x_0:

      i. $x_0 = x_1 - x_2$. Solution at: $(10.2,-7)$, value 17.2.

      ii. $x_0 = -x_1 + x_2$. Solution at: $(3,5)$, value 2.

      iii. $x_0 = 3x_1 + 4x_2$. Solution at: $(3,5)$, value 29.
      The solutions are unique.

   On the other form, the feasible set is a triangle with vertices $(2,4)$, $(0,-4)$, and $(8.4,-4)$. The solutions, respectively, are: $(8.4,-4)$, value 12.4; $(2,4)$, value 2; and $(2,4)$, value 22.

2. This problem concerns the linear programming problem from question 1, with $x_0 = x_1 + x_2$.

(a) Write the problem in the form $\max c \cdot x$ subject to $Ax \leq b, x \geq 0$.

On one form:

$$\begin{array}{lrcrcrcrcr}
\text{max} & u_1 & - & v_1 & + & u_2 & - & v_2 & & \\
\text{subject to} & 5u_1 & - & 5v_1 & + & 3u_2 & - & 3v_2 & \leq & 30 \\
 & -5u_1 & + & 5v_1 & + & u_2 & - & v_2 & \leq & -10 \\
 & & & & & -u_2 & + & v_2 & \leq & 7
\end{array}$$

On the other:

$$\begin{array}{lrcrcrcrcr}
\text{max} & u_1 & - & v_1 & + & u_2 & - & v_2 & & \\
\text{subject to} & 5u_1 & - & 5v_1 & + & 4u_2 & - & 4v_2 & \leq & 26 \\
 & -4u_1 & + & 4v_1 & + & u_2 & - & v_2 & \leq & -4 \\
 & & & & & -u_2 & + & v_2 & \leq & 4
\end{array}$$

(b) Write the problem in the form $\max c \cdot x$ subject to $Ax = b, x \geq 0$.

$$\begin{array}{lrcrcrcrcrcrcrcr}
\text{max} & u_1 & - & v_1 & + & u_2 & - & v_2 & & & & & & & & \\
\text{subject to} & 5u_1 & - & 5v_1 & + & 3u_2 & - & 3v_2 & + & s_1 & & & & & = & 30 \\
 & -5u_1 & + & 5v_1 & + & u_2 & - & v_2 & & & + & s_2 & & & = & -10 \\
 & & & & & -u_2 & + & v_2 & & & & & + & s_3 & = & 7
\end{array}$$

On the other form:

$$\begin{array}{lrcrcrcrcrcrcrcr}
\text{max} & u_1 & - & v_1 & + & u_2 & - & v_2 & & & & & & & & \\
\text{subject to} & 5u_1 & - & 5v_1 & + & 4u_2 & - & 4v_2 & + & s_1 & & & & & = & 26 \\
 & -4u_1 & + & 4v_1 & + & u_2 & - & v_2 & & & + & s_2 & & & = & -4 \\
 & & & & & -u_2 & + & v_2 & & & & & + & s_3 & = & 4
\end{array}$$

(c) Write the dual of the problem.

$$\begin{array}{lrcrcrcr}
\text{min} & 30y_1 & - & 10y_2 & + & 7y_3 & & \\
\text{subject to} & 5y_1 & - & 5y_2 & & & \geq & 1 \\
 & -5y_1 & + & 5y_2 & & & \geq & -1 \\
 & 3y_1 & + & y_2 & - & y_3 & \geq & 1 \\
 & -3y_1 & - & y_2 & + & y_3 & \geq & -1
\end{array}$$

and the other form:

$$\begin{array}{lrcrcrcr}
\text{min} & 26y_1 & - & 4y_2 & + & 4y_3 & & \\
\text{subject to} & 5y_1 & - & 4y_2 & & & \geq & 1 \\
 & -5y_1 & + & 4y_2 & & & \geq & -1 \\
 & 4y_1 & + & y_2 & - & y_3 & \geq & 1 \\
 & -4y_1 & - & y_2 & + & y_3 & \geq & -1
\end{array}$$

In all of the above problems, it is understood that all variables that appear in the problem are constrained to be nonnegative.

3. Consider the linear programming problem: $\max 2x_1 - x_2$

$$\begin{array}{lrcrcr} \max & 2x_1 & - & x_2 & & \\ \text{subject to} & x_1 & + & 3x_2 & \leq & 30 \\ & 2x_1 & - & 3x_2 & \leq & 20 \\ & & & x & \geq & 0 \end{array}$$

(a) Write the initial simplex array for the problem. (That is, write the problem in a form suitable for a simplex algorithm pivot.)

(b) Make one simplex algorithm pivot using the array you provided in part (a). (If it is not possible to make a pivot explain why not. If it is possible to make a pivot, state the guess for the problem provided by your pivot, and state whether this guess solves the optimization problem.)

Use the tables below (which may contain extra rows and or columns) for your answers.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	-2	1	0	0				0
(1)	x_3	1	3	1	0				30
(2)	x_4	$< 2 >$	-3	0	1				20
(3)									

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	-2	0	1				20
(1)	x_3	0	4.5	1	$-.5$				20
(2)	x_1	1	-1.5	0	.5				10
(3)									

This is not a solution because there is still a negative number in row 0. The guess is: $x_0 = 20, x_1 = 10, x_2 = 0, x_3 = 20, x_4 = 0$.

For the other form:

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	-2	1	0	0				0
(1)	x_3	$< 2 >$	3	1	0				20
(2)	x_4	1	-3	0	1				30
(3)									

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	4	1	0				20
(1)	x_1	1	1.5	.5	0				10
(2)	x_4	0	-4.5	$-.5$	1				20
(3)									

This is a solution because there are no negative numbers in row 0. The solution is: $x_0 = 20, x_1 = 10, x_2 = 0, x_3 = 0, x_4 = 20$.

4. Which of the tables below correspond to arrays that could arise in a correct simplex algorithm computation? The objective is to maximize x_0 and all of the variables in the problem are constrained to be nonnegative.

   The question was the same, but on the other form the order was different.

   (a) is not (negative number in value column); (d) is not (x_1 is not a basis element). The others are.

   (On the other form (e) is the one with a negative number in the value column and (c) is the one where x_1 is not in the basis.)

5. The questions were the same, but the order was different.

   The basic ideas are that if two points are in the feasible set, then the segment connecting them must also be in the feasible set, and feasible sets have line segments for boundaries. Hence the possibilities are (a) and (c) ((b) and (e) on the other form).

6. The questions were the same on the other form, but the order was somewhat different.

   For each of the statements below, circle **TRUE** if the statement is always true, circle **FALSE** otherwise. No justification is required.

   These problems refer to the linear programming problem (P) written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

   and its dual

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0.$$

   (a) **TRUE** **FALSE**
   If (D) is not feasible, then (P) is unbounded.
   This is false. (P) may be infeasible.

   (b) **TRUE** **FALSE**
   Let u be a vector of ones (with the same number of components as x. If (P) has a solution, then

$$\max c \cdot (x - u) \text{ subject to } Ax \leq b, x \geq 0$$

   has a solution.
   True. (P) remains feasible (the feasible set doesn't change). (D) also remains feasible (the feasible set gets larger, since all of the right-hand sides get smaller).

   (c) **TRUE** **FALSE**
   If (P) has a solution and $\overline{c} \geq c$, then

$$\max \overline{c} \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

   has a solution ($\overline{c}$ may be different from c).
   This is false. The problem remains feasible, but could be unbounded. A dumb example is the pair of unconstrained problems: $\max c \cdot x$ and $\max \overline{c} \cdot x$, where $c = 0$ (so everything is a solution and the value is zero) and $\overline{c} = 1$ (so that the problem is unbounded).

   (d) **TRUE** **FALSE**

   If a linear programming problem is infeasible, then it will continue to be infeasible if the objective function changes.
   This is true. Changing the objective function does not change the feasible set.