Econ 172A, W2002: Midterm Examination I

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.
2. The examination has 6 questions. Answer them all.
3. If you do not know how to interpret a question, then ask me.
4. You must justify your answers to the first three questions. No justification is needed on the last three questions.
5. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		30
II		30
III		20
IV		20
V		20
VI		20
Exam Total		140

1. Consider the linear programming problem:

$$\max x_0$$

$$\begin{array}{rrcrcr} \text{subject to} & 5x_1 & + & 3x_2 & \leq & 30 \\ & 5x_1 & - & x_2 & \geq & 10 \\ & & & x_2 & \geq & -7 \end{array}$$

   (a) Graphically represent the feasible set of this problem.

   (b) Graphically solve the problem for the following values of x_0:

      i. $x_0 = x_1 - x_2$.

      ii. $x_0 = -x_1 + x_2$.

      iii. $x_0 = 3x_1 + 4x_2$.

   In each case, graphically identify the solution; write down the values for x_1 and x_2 that solve the problem; write down the value of the problem. If the solution to the problem is not unique, then give two solutions.

2. This problem concerns the linear programming problem from question 1, with $x_0 = x_1 + x_2$.

   (a) Write the problem in the form $\max c \cdot x$ subject to $Ax \leq b, x \geq 0$.

   (b) Write the problem in the form $\max c \cdot x$ subject to $Ax = b, x \geq 0$.

   (c) Write the dual of the problem.

3. Consider the linear programming problem:

$$\begin{array}{lrcrcr} \max & 2x_1 & - & x_2 & & \\ \text{subject to} & x_1 & + & 3x_2 & \leq & 30 \\ & 2x_1 & - & 3x_2 & \leq & 20 \\ & & & x & \geq & 0 \end{array}$$

(a) Write the initial simplex array for the problem. (That is, write the problem in a form suitable for a simplex algorithm pivot.)

(b) Make one simplex algorithm pivot using the array you provided in part (a). (If it is not possible to make a pivot explain why not. If it is possible to make a pivot, state the guess for the problem provided by your pivot, and state whether this guess solves the optimization problem.)

Use the tables below (which may contain extra rows and or columns) for your answers.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0								
(1)									
(2)									
(3)									

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0								
(1)									
(2)									
(3)									

4. Which of the tables below correspond to arrays that could arise in a correct simplex algorithm computation? The objective is to maximize x_0 and all of the variables in the problem are constrained to be nonnegative. I did not include a column for the variable x_0. (Each part of the question is independent from the other parts.) [To answer the question, simply circle the letter (or letters) corresponding to correct simplex arrays.]

(a)

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	1	−1	2	0	0	−2	0	6
(1)	x_7	−1	1	2	0	0	1	1	6
(2)	x_5	0	2	1	0	1	−1	0	−4
(3)	x_4	0	−1	0	1	0	1	0	10

(b)

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	−1	2	0	0	−2	0	6
(1)	x_1	1	1	2	0	0	1	1	6
(2)	x_5	0	2	1	0	1	−1	0	4
(3)	x_4	0	−1	0	1	0	1	0	10

(c)

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	−1	2	0	0	−2	0	6
(1)	x_7	1	−1	2	0	0	1	1	6
(2)	x_5	0	0	1	0	1	−1	0	4
(3)	x_4	0	−1	0	1	0	1	0	10

(d)

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	−1	2	0	2	−2	1	6
(1)	x_1	−1	1	2	0	0	1	1	6
(2)	x_5	0	2	1	0	1	−1	0	4
(3)	x_4	0	−1	0	1	0	1	0	10

(e)

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	1	0	0	0	2	1	6
(1)	x_1	1	1	2	0	0	1	1	6
(2)	x_5	0	2	1	0	1	−1	0	4
(3)	x_4	0	−1	0	1	0	1	0	10

5. Which of the following can be a feasible set for a linear programming problem?

(a)

(b)

(c)

(d)

(e)

6. For each of the statements below, circle **TRUE** if the statement is always true, circle **FALSE** otherwise. No justification is required.

   These problems refer to the linear programming problem (P) written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

   and its dual

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0.$$

   (a) **TRUE** **FALSE**
   If (D) is not feasible, then (P) is unbounded.

   (b) **TRUE** **FALSE**
   Let u be a vector of ones (with the same number of components as x. If (P) has a solution, then

$$\max c \cdot (x - u) \text{ subject to } Ax \leq b, x \geq 0$$

   has a solution.

   (c) **TRUE** **FALSE**
   If (P) has a solution and $\bar{c} \geq c$, then

$$\max \bar{c} \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

   has a solution ($\bar{c}$ may be different from c).

   (d) **TRUE** **FALSE**

   If a linear programming problem is infeasible, then it will continue to be infeasible if the objective function changes.