

## Econ 172A, Winter 2002: Problem Set 2

**Instructions:** Due: February 26, 2002, in class (no late papers). Please supply complete answers. When you generate sensitivity reports using Excel, make sure that you check “assume linear model” (solver option). Otherwise, you will get decimal answers and weird things (like Lagrange multipliers) in your sensitivity report. You should be able to do all but the last part of the second problem now. I will discuss how to interpret the sensitivity reports (needed to answer questions 1 and 3) in lecture on February 12 and 14 and complementary slackness in the following week.

1. I attach the sensitivity and answer report from a linear programming problem. Start at the top, go from left to right, identify any numerical information that you can figure out from information that comes before it. When you reach a cell that can be figured out by previous information, identify the cell, and explain how you could have figured out the value. For example, the first “redundant” cell is the reduced cost of the variable **WHOLE**. You know that this number is 0 because the reduced cost is always zero when a variable takes on a positive value (and you know that the variable takes the value 5).
2. Professor Foster’s family owns 125 acres of land (“Foster Farms”) and has \$40,000 in funds available to invest in the farm. The family works on the farm. They can supply a total of 3,500 person-hours of labor during the winter months and 4,000 person-hours during the summer. Any labor that is not used farming can be used to do odd jobs (like teaching Econ 120). These jobs pay \$5.00 per hour during the winter and \$6.00 per hour during the summer. Cash income comes from three crops, soybeans, corn, and oats, and two types of livestock, cows and hens. No investments are needed for the crops. On the other hand, cows require an investment of \$1,200 each, while hens cost \$9 each. Each cow requires 1.5 acres of land, 100 person-hours of work during the winter and 50 hours of person-hours during the summer. Each cow produces an annual cash income of \$1,000. Similarly, for each hen the farm earns \$5, requires no acreage, but uses .6 person-hours during the winter and an additional .3 hours during the summer. The farm has a chicken house that has capacity of 3,000 hens and a barn that can hold up to 32 cows. The table below provides relevant information on the input requirements and profitability of the crops. The Foster family wishes to determine how much acreage should be planted in each of the crops and how many cows and hens should be kept to maximize its net cash income.

	Soybeans	Corn	Oats
Winter person-hours	20	35	10
Summer person-hours	50	75	40
Net annual cash income (\$)	500	750	350

- (a) Formulate a linear program that describes the Foster family's problem.
- (b) Write the dual of the problem.
- (c) Interpret the dual.
- (d) Solve the primal and the dual using excel.
- (e) Compare the answers and confirm the conclusion of the duality theorem of linear programming and all complementary slackness conditions.

3. This is the first practice formulation problem:

A nut packer has on hand 150 pounds of peanuts, 100 pounds of cashews, and 50 pounds of almonds. The packer can sell three kinds of mixtures of these nuts: a cheap mix consisting of 80% peanuts and 20% cashews; a party mix with 50% peanuts, 30% cashews, and 20% almonds; and a deluxe mix with 20% peanuts, 50% cashews, and 30% almonds. If the 12-ounce can of the cheap mix, the party mix, and the deluxe mix can be sold for \$.90, \$1.10, and \$1.30 respectively, formulate a linear programming problem that determines the number of cans of each type the producer should produce to maximize her return.

You may formulate this problem yourself or look up the answer (it is posted with the solutions to the practice formulation problems).

- (a) Solve the problem using Excel. (Include a clearly labeled spreadsheet and indicate the solution to the problem and the value of the problem.)  
Use the Excel answer and sensitivity reports to answer as many of the following questions as possible. If you cannot answer the question using these reports, use Excel and resolve the problem.
- (b) What happens to the solution and to profits if the price of cheap mix goes up to \$1.00 per can?
- (c) What happens to the solution and to profits if the price of cheap mix falls to 50 cents per can?
- (d) What would happen to the profits of the nut packer if she lost 20 pounds of peanuts?
- (e) How much would the nut packer be willing to pay for an extra 10 pounds of almonds?
- (f) Imagine a new product called alshews that is 50% almonds and 50% cashews. What would the price of a 12-ounce can of alshews need to be for it to be profitable (relative to the nut packer's other options) to sell them?
- (g) Suppose that the nut packer was prohibited from selling more than 100 cans of cheap mix. How would the solution and profit change?

- (h) Suppose that the nut packer had to sell at least 100 cans of party mix. How would the solution and profit change?
- (i) Suppose that the nut packer receives 100 pounds of a miracle nut that can replace any kind of nut in any of the mixtures. How would this change her profit and production plans?