Econ 172A, W2002: Final Examination

Instructions.

1. The examination has six questions. Answer them all.

2. If you do not know how to interpret a question, then ask me.

3. You must justify your answers to the questions 2 through 5. No justification is necessary for questions 1 and 6.

4. The table below indicates how points will be allocated on the exam.

5. Work alone. You may not use notes or books.

6. You have three hours.

7. I will leave graded examinations in Sequoia 245.

	Score	Possible
I		27
II		26
III		24
IV		30
V		36
VI		32
Exam Total		175
Course Total		350
Grade in Course		

1. I attach the sensitivity and answer report from a linear programming problem. Start at the top, go from left to right, identify any numerical information that you can figure out from information that comes before it. When you reach a number that can be figured out by previous information, circle the cell. You will get one point for each number you correctly circle. You will lose one point for each number you incorrectly circle. You will get a bonus if your answer is completely correct.

2. Von's Grocery Store is reorganizing its store in University City. The manager has yet to decide where to put four different items, canned soup, bread, beans, and candy. There is space in four aisles. Extensive market research permits the manager to estimate the annual profit (numbers below are in thousands of dollars) for each possible product placement. The table below contains this information.

	1	2	3	4
Soup	10	6	12	8
Bread	15	18	5	11
Beans	17	10	13	16
Candy	14	16	6	12

(a) Assume that there is room for exactly one item in each aisle and that items can be placed in no more than one location. Find an assignment of foods to aisles that maximizes the total expected profit. Your answer should describe where to locate each food and the associated profit. You should explain how you arrived at the answer and why it solves the problem.

(b) Assume that the store now has the opportunity to sell another item, magazines. Selling magazines will lead to profit of $20,000 per year no matter where they are placed. However, the store is still able to put only one item in each aisle, so if the manager decides to sell magazines, then she will be unable to sell one of the other items. As in the first part, assume that items can be placed in no more than one location. Set up the problem you must solve in order to find an assignment of foods and magazines to aisles that maximizes the total expected profit. You are not required to identify the profit maximizing location of items for this problem, but you must clearly describe what problem the manager must solve in order to maximize profit, what method you would use to solve the problem, and why the method would work.

3. More than two hundred years ago the French mathematician de Montmort proposed the following gift to his son. "I shall put a gold coin into either my right hand or my left hand. You will name a hand. If your guess is correct and the coin is in my right hand, you will get the gold coin. If your guess is correct and the coin is in my left hand, you shall receive two gold coins. Otherwise, you will get nothing." Assume that de Montmort and his son play this game, that de Montmort wants to minimize the amount of gold that he expects to pay to his son and his son wants to maximize the amount of gold he wins.

   (a) Write the payoff matrix for this game. Clearly label the strategies and explain how you computed the payoffs.

   (b) Find the (pure-strategy) security levels for both players. Would it be sensible to use a pure strategy in this game? Explain.

   (c) Are there any dominated strategies in the game? Identify them and explain why they are dominated.

   (d) Find the mixed strategy security level of the game.

   (e) What is the value of the game and what are the equilibrium strategies?

   (f) Suppose now that the son is able to see which hand de Montmort hides the coin before the son decides which hand to guess. (De Montmort knows that his son knows where the coin will be hidden.) Write the payoff matrix for this version of the game. Answer questions (c) and (e) for this version of the game.

4

4. A company can produce two products. The table below summarizes the production technology. Each week, up to 400 units of raw material can be purchased at a cost of $1.50 per unit. The company employs four workers, who work 40 hours per week. The base salaries of the workers are considered a fixed cost and do not enter the computation. Workers are paid $6 per hour to work overtime. Each week, 320 hours of machine time are available.

	Product 1	Product 2
Selling Price	$15	$8
Labor Required	.75 hour	.5 hour
Machine Time Required	1.5 hour	.8 hour
Raw Material Required	2 units	1 unit

If the firm does not advertise, 50 units of product 1 and 60 units of product 2 will be demanded each week. Advertising can be used to stimulate demand. Each dollar spent on advertising Product 1 increases the demand for Product 1 by 10 units. Each dollar spent on advertising Product 2 increases the demand for Product 2 by 15 units. At most $100 can be spent on advertising.

To formulate the problem, I defined the following variables:

P_1 = the number of units of Product 1 produced each week.

P_2 = the number of units of Product 2 produced each week.

OT = the number of hours of overtime labor used each week.

RM = the number of units of raw material purchased each week.

A_1 = the amount (in dollars) spent each week advertising Product 1.

A_2 = the amount (in dollars) spent each week advertising Product 2.

The firm's optimization problem is then:

$$
\begin{array}{llllllllll}
\max & 15P_1 & + & 8P_2 & - & 6OT & - & 1.5RM & - & A_1 & - & A_2 \\
\text{subject to} & P_1 & & & & & & & - & 10A_1 & & & \leq & 50 \\
& & & P_2 & & & & & & & - & 15A_2 & \leq & 60 \\
& .75P_1 & + & .5P_2 & - & OT & & & & & & & \leq & 160 \\
& 2P_1 & + & P_2 & & & - & RM & & & & & \leq & 0 \\
& & & & & & & RM & & & & & \leq & 400 \\
& & & & & & & & & A_1 & + & A_2 & \leq & 100 \\
& 1.5P_1 & + & .8P_2 & & & & & & & & & \leq & 320
\end{array}
$$

$$P_1, P_2, OT, RM, A_1, A_2 \geq 0.$$

I solved this problem using Excel. The output follows this problem. Use the output to answer the questions below. Answer the questions independently (so that a change described in one part applies only to that part).

Answer as many questions below as you can using the available information. If there is some question that you cannot answer using the output that I have provided, explain why not and say as much as you can using the available information.

(a) If overtime costs $4 per hour, would the company use it?

(b) If each unit of Product 1 sold for $15.50 would the current basis remain optimal? What would be the new solution?

(c) What is the most that the company would be willing to pay for another unit of raw material?

(d) How much would the company be willing to pay for another hour of machine time?

(e) If each worker were required (as part of the regular work week) to work 45 hours per week, what would the company's profit be? (That is, assume that the company gets 45 hours per week from a worker without paying overtime.)

(f) How would the solution of the problem change if you could not use more than 20 hours of machine time per week in the production of the first product?

(g) How would the firm's profits change if advertising for Product 1 was free?

(h) Suppose that the technology was improved so that only .5 hours of labor were needed to produce one unit of Product 1. How would the answer change?

5. The California Cheese Company produces two cheese spreads by blending mild cheddar cheese with sharp cheddar cheese. The cheese spreads are sold in one pound containers. There are two blends. Regular blend contains 80% mild cheddar and 20% sharp cheddar. The tangy blend contains 60% mild cheddar and 40% sharp cheddar. The company can buy as much as 8100 pounds of mild cheddar cheese for $1.20 per pound and up to 3000 pounds of sharp cheddar cheese for $1.40 per pound. The cost to blend and package the cheese spreads is $.20 per one-pound container. (This cost does not include the cost of the cheese.) Each container of the regular blend sells for $2.60 and each container of tangy sells for $2.20. The California Cheese Company wants to determine how many containers of each blend to produce. Its goal is to maximize profits.

In order to formulate its problem, the company defines the following variables:

R = number of containers of regular blend produced.

T = number of containers of tangy blend produced.

M = number of pounds of mild cheddar used.

S = number of pounds of sharp cheddar used.

Use these variables and the information about the problem given above to provide the information requested.

(a) Write an expression for the total cost of the company's production plan.

(b) Write an expression for the total revenue of the company's production plan.

(c) Write expressions that correctly describe the availability of mild and sharp cheddar cheeses.

(d) Write down any other constraints needed to completely describe the optimization problem of the cheese company.

(e) Write the linear programming problem that describes the company's optimization problem. Use only the expressions you wrote as answers to the previous parts of the question.

6. In each part indicate which of the choices are correct. (For each part, there may be 0, 1, 2, 3, or 4 correct choices.)

   (a) The inequalities below describe two-dimensional sets. Which of these sets can be described by linear inequalities?

       i. $x + y^2 \leq 4$.

       ii. $x + y \leq 4$.

       iii. $|x + y| \leq 4$.

       iv. $\frac{x}{x+y+2} \leq 4, x \geq 0, y \geq 0$.

   (b) Which of the following must be true in any valid simplex algorithm array with at least one constraint?

       i. All entries in the value column (excluding the value of x_0) are non-negative.

       ii. There is at least one non-negative number in each column.

       iii. There is at least one zero in each row.

       iv. There is at least one negative number in row 0.

   (c) Which of the statements below are true statements about following linear programming problem (P) or its dual (D):

$$
\begin{array}{lrcrcrcrcr}
\max & 15x_1 & + & 6x_2 & + & 9x_3 & + & 2x_4 & & \\
\text{subject to} & 2x_1 & + & x_2 & + & 5x_3 & + & 6x_4 & \leq & 20 \\
& 3x_1 & + & x_2 & + & 3x_3 & + & 3x_4 & \leq & 24 \\
& 7x_1 & & & & & + & x_4 & \leq & 70 \\
& & & & & & & x & \geq & 0
\end{array}
$$

       i. $x^* = (4, 12, 0, 0)$ is a solution to (P).

       ii. $x^* = (10, 6, -4, 0)$ is a solution to (P).

       iii. $y^* = (3, 3, 0)$ is a solution to (D).

       iv. If the right-hand side of the third constraint increased from 70 to 85, the solution to (P) would not change.

   (d) Consider the linear programming problem:

$$
\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0.
$$

   Assume that all entries in A, b, and c are whole numbers and that the problem has a solution x^*. By a whole number solution, I mean a solution x^{**} with the property that every component of x^{**} is a whole number.

       i. The problem must have a whole number solution.

       ii. The problem must have a whole number solution provided that all of the entries in A, b, and c are equal to -1, 0, or 1.

       iii. The problem must have a whole number solution provided that the dual of the problem has a whole number solution.

       iv. If the problem has a whole number solution, then the value of the problem is a whole number.