

**Econ 172A, Fall 2003: Problem Set 3**

**Instructions:** Due: December 4, 2003, in class (no late papers).

1. For the following two-person, zero-sum games, find the value and the equilibrium strategies.

(a) 

5	5
2	4

(b) 

0	5
2	4

(c) 

0	5	2
2	4	8

(d) 

0	5	2
2	4	0

(e) 

0	1	6
6	4	0

2. Consider the two-player zero-sum game:

5	10	7	4
6	3	7	15
15	20	8	10
6	15	7	2

- (a) Find the equilibrium strategies and the value. (You could do this by hand, but I would prefer if you tried to get the answer using Excel.)
- (b) How does the answer change if all the entries in the first row of the payoff matrix doubled? Is this change to the advantage of player one? Explain.
- (c) How does the answer change if all the entries in the payoff matrix decreased by 5?
- (d) How does the answer change if the largest element increases from 20 to 25?
- (e) To what extent are the answers to parts (b)-(d) general? Explain.

You can obtain answers to parts (b) and (c) (and get some intuition on (d)) by solving variations of the original problem using excel.

3. Colonel Blotto commands two companies, and his enemy, Captain Shazzam, has only one. Each side tries to capture the opposing camp without losing his own. The two commanders must decide simultaneously how many companies to use to attack the enemy camp. If the defenders of a camp are outnumbered by the attackers (that is, there are more attacking companies than defending companies), then the camp is captured. Otherwise, the result is a standoff. Companies that are not sent to attack the enemy defend the home camp. A side wins (receives a payoff 1) if it captures the other's camp without losing its own. If neither side wins, both sides receive the payoff zero. The losing side receives a payoff of  $-1$ .

- (a) Write the payoff matrix for this game. Clearly label the strategies and explain how you computed the payoffs.

- (b) Find the pure-strategy security levels for both players. Would a rational player use a pure strategy in this game? Explain.
- (c) Are there any dominated strategies in the game? Identify them and explain why they are dominated.
- (d) Find the mixed-strategy security level of the game.
- (e) Suppose that Captain Shazzam has a spy that reports (with complete accuracy) how many companies Colonel Blotto plans to use in his attack. How does this change influence the game? Write a game that describes the new situation and find the value of the game. (You may make simplifying assumptions to describe the game. Please describe these assumptions explicitly and explain why they are justified.)