Econ 172A, Fall 2003: Problem Set 2

Instructions: Due: November 11, 2003, in class (no late papers). Please supply complete answers. When you generate sensitivity reports using Excel, make sure that you check "assume linear model" (solver option). Otherwise, you will get decimal answers and weird things (like Lagrange multipliers) in your sensitivity report.

1. The Pallo Winery produces three varieties of wine – red, white, and blue. These products sell for $15, $20, and $25 per case, respectively. Each case of red wine costs $10 to produce and requires one hour to process and six hours to bottle. Each case of white wine costs $12 to produce, needs two hours of processing time and eight hours of bottling time. Blue wine costs $21 per case to produce, uses three hours of processing time, and eight hours in bottling time. Each week there are 220 hours of processing time and 480 hours of bottling time available in the Winery's factories.

   Pallo stores its wine in a warehouse with a capacity of 3000 cubic feet. Each week they ship out their entire inventory. White and red wine occupy 1.5 cubic feet per case. The blue wine occupies two cubic feet per case. Pallo has a contract with a local wine seller to deliver at least 60 cases of red wine each week. Formulate a linear programming problem that determines the number of cases of each type of wine to produce, assuming that Pallo wants to maximize profit. Solve this problem using Excel.

2. Use your solution to the first problem to answer the following questions. You should try to answer as many parts as possible using the sensitivity analysis output from Question 1. If you need to resolve the problem, then include the relevant Excel worksheet (or whatever supplementary computations you did to solve the problem). Treat each of the parts below independently.

   (a) What happens to Pallo's production and profits if the price of red wine goes up by $1 per case?

   (b) What happens to Pallo's production and profits if the production cost of blue wine doubles?

   (c) What happens to Pallo's production and profits if the production cost of blue wine is cut in half?

   (d) What happens to Pallo's production and profits if the price of white wine falls to $18 per case?

   (e) What happens to Pallo's profits if it were to have only 100 hours of processing time available?

   (f) What happens to Pallo's profits if it were to have only 70 hours of processing time available?

   (g) Pallo gets the opportunity to buy an empty warehouse. How much would Pallo pay for the warehouse? How does the payment depend on the capacity of the warehouse?

   (h) Demand for red wine falls and Pallo renegotiates its contracted delivery obligations. Now it needs only supply 50 cases of red wine each week. How do profits change?

   (i) What happens to Pallo's profits if the time it takes to bottle white wine falls by 1 hour per case?

   (j) A member of the Pallo family invents yellow wine. Yellow wine costs $5 per case to produce, can be sold for $8 per case, occupies 10 cubic feet per case, needs two hours of processing time, and requires 2 hours per case to bottle. Would producing yellow wine increase Pallo's profits?

3. Consider the linear programming problem:

$$
\begin{array}{rrrrrrr}
\max & 3x_1 & + & x_2 & + & x_3 & \\
\text{subject to} & -x_1 & & & + & 5x_3 & \geq & 50 \\
& 2x_1 & - & 3x_2 & & & \leq & 8 \\
& x_1 & & x_2 & & & \geq & 0
\end{array}
$$

1

(a) Solve the problem using Excel.

(b) Write the problem as a maximization problem, subject to all variables being nonnegative, and constraints of the form "something $\leq$ constant."

(c) Find the dual of the problem (in order to do this, the form that you fould in part b should be useful).

(d) Solve the problem in part b using Excel.

(e) Compare the solutions you found in parts a and d. In what ways are they different? In what ways are they the same? Compare the solution to the dual problems (you need the sensitivity report, where dual variables are called Shadow Prices).

Hand in the output with a copy of the problem and the solution and variables plainly labeled.

4. I solved a version of a linear programming problem using Excel. I attach the answer report and the sensitivity report from Excel. (I deleted a few irrelevant columns.) In these reports, I replaced several values with question marks (???). Your job is to replace these question marks with the correct information. In the "Status" column, the correct information is either the word "Binding" or the words "Not Binding;" otherwise, you need to supply a number. I have not given you enough information to reconstruct the problem. You should fill in the missing values using your knowledge of Excel, duality theory, and complementary slackness. You may not have sufficient information to complete the table. If you cannot determine some of the missing numbers, then say so. If you can fill in a value, then explain what permitted you to do so. Can you determine the final value of the problem? If so, what is it?