Econ 172A, Fall 2003: Final Examination Possible Answers

Here are suggested answers to the final. There were 250 possible points on the exam. The median was 165.

1. (a) The constraint describes how much of the first product you can sell [2]. You can sell 50 units if you don't advertise [2] and 10 more units for each dollar you spend on advertisements [2].
   (b) The change causes the OT coefficient in the objective function to go up by two [2]. Since this is in the allowable range [2], the solution does not change [2].
   (c) No, increasing the coefficient of product 2 in the objective function by .5 changes the optimal basis because it is outside the allowable range [3]. The solution does change [1]. Profit goes up more 50 cents times the number of units of product two produced, a total of 40. New profit at least: $2467.67 [2].
   (d) The dual price for the resource constraint (row 5) is 6 [2]. This is in allowable range [2]. Hence profit goes up by 6 [2]. Note that the dual price for the constraint in row 6 is 4.5. The dual variable for that constraint measures the value of having another unit of resource available. It is 4.5 rather than 6 because it assumes that you must pay ($1.50) for it. Since I stated that the additional resource arrived at no charge, $6 is the increase in profit. (give 2 points for $4.50 if they provide an almost correct reason)
   (e) There is slack in this constraint, so the firm would pay nothing for more machine time (constraint 8) [6].
   (f) You are given 20 extra hours [1]. This change raises the right hand side of constraint 3 from 160 to 180 [1]. An increase in 20 is within the allowable range [1]. Hence the old basis remains optimal. Profit goes up by 20 times the dual price of the labor contraint [1]. That is, profit goes up by $77.33 [1]. The new profit is $2505 [1].
   (g) No. The resources used are: 3 units of raw material, which is worth $6 per unit; 1.5 hours of labor, which is worth $3.87; and some machine time, which is worth nothing, since there is slack in that constraint [8: for realizing that you can use shadow prices to compute the value of the ingredients needed to create the new product]. Hence the resource cost is $18 + $3.87 = $21.87 per unit [2], which is more than the profit contribution of the product [2].
   (h) Yes. The invention frees up 160(.05) = 8 units of raw material [4]. The allowable increase for raw material is 6.6667 [2]. Hence the firm increases profits by at least 6 (the dual price of raw material) [2] times 6.66667 or 38 [4].

2. (a)

	FAST	CURVE
TAKE	-1	1
SWING	2	-1

(b) No.

(c) No

(d) ROW: -1; COLUMN: 1 (3 points each; some explanation needed)

(e) Value is .2; Row "takes" with probability .6 and "swings" with probability .4; Column "fast" with probability .4 and curve with probability .6. Four points for each player's strategy and two points for the value. Computation should show understanding of what to do.

(f) $1.25 = 2(.75) - 1(.25) > -1(.75) + (.25) = -.5$, so it is better to swing and the payoff is 1.25. [Give 4 points for understanding what computation to do and 2 points for doing it correctly.]

(g) Since $-.2 = 1(.4) - 1(.6) < -1(.4) + 2(.6)$, Column should throw a curve. She expects to win .2. [Grade like the previous part.]

(h)

	FAST	CURVE/FAST	CURVE/CURVE
TAKE/SWING	-1	-1	1
TAKE/TAKE	-1	2	1
SWING	2	-1	-1

The important point to note is that the strategies must specify what to do when there is the possibility of two pitches. If the pitcher starts with a fast ball, then there is no need for her to plan on a second pitch. Otherwise, the strategy must specify what the second pitch is. Similarly, if the batter gets a second chance to hit (because at the first pitch he did not swing at a curve), then there are two possible things to do for the second pitch. Students may use a larger payoff matrix, but students who failed to understand that there are more than two strategies should receive little credit.

3. Since the Column player has only two strategies, I would use a graph to compute her security level. This yields the strategy of playing left with probability .5 and right with probability .5 and the value (for Row) of 4. Using these strategies, the middle strategy of Row is irrelevant, so you can find the Row player's best strategy using just the top two strategies. This turns out to be $(.5, 0, .5)$. You can compute these strategies graphically or by solving for the mixing probability that leads to the relevant two pure strategies creating the highest payoffs. It is true that the middle strategy is dominated – but a mixture of the top and bottom strategies. Students who explicitly pointed out this dominance received full credit, but we deduct five points for people who asserted dominance with no justification.

   Students receive 10 points for correctly identifying pure strategy security levels and pointing out that this is not a solution. If they do not do this, give 10 points for correctly drawing a picture that will lead to the identification of an equilibrium mixed strategy, 5 more points for correctly identifying the equilibrium strategy of Column, 5 points for the value, and

5 points for Row's strategy. If they first find security levels, give 6 points for the picture, and 3 each for value, Column strategy, and Row strategy.

4. There are many answers. Here is one:

$$\begin{array}{lrcrcr} \text{max} & x_1 & + & x_2 & & \\ \text{subject to} & x_1 & + & x_2 & \leq & 2 \\ & -x_1 & - & x_2 & \leq & -3 \\ & & & x & \geq & 0 \end{array}$$

The first constraint says that $x_1 + x_2 \leq 2$, while the second says that $x_1 + x_2 \geq 3$. So they are obviously inconsistent. The dual is:

$$\begin{array}{lrcrcr} \text{min} & 2y_1 & - & 3y_2 & & \\ \text{subject to} & y_1 & - & y_2 & \geq & 1 \\ & y_1 & - & y_2 & \geq & 1 \\ & & & y & \geq & 0 \end{array}$$

The dual is clearly feasible ($y_1 = 1, y_2 = 0$), so the duality theorem says that it is unbounded. More directly, notice that if you want the objective function of the dual to be small let $y_1 = 1 + K$ and $y_2 = K$. This yields a value of $2 - K$ which can be made arbitrarily small by increasing K.

Grading notes: 30 points total. Students get 4 points for writing down any linear programming problem in the requested form with two constraints and two variables. They get 4 more points if the problem is infeasible and they clearly state that it is infeasible. They get 4 more points for providing a justification (a graph, manipulating constraints, etc). They get 4 more points for showing evidence of thinking about the dual to their problem (for example, by writing it down). They get 7 points if the dual is unbounded and 7 more for an explanation. Students who write down an infeasible problem, verify infeasibility, and examine the dual should get at least 16. You can give additional partial credit if they say anything else that is reasonable. For example, if their dual is infeasible and they say so, I'd give another 3-5 points. On the other hand, give only 16 points for students who simply reproduce last year's solution (last year I required students to provide an infeasible primal with an infeasible dual).

5. There are many approaches. I prefer writing down the dual and using complementary slackness. The dual is:

$$\begin{array}{lrcrcr} \text{min} & 2y_1 & + & y_2 & & \\ \text{subject to} & y_1 & - & y_2 & \geq & 2 \\ & 4y_1 & + & 2y_2 & \geq & -4 \\ & 8y_1 & + & 4y_2 & \geq & -6 \\ & -2y_1 & + & 3y_2 & \geq & 5 \\ & & & y & \geq & 0 \end{array}$$

Plugging in the suggested values for (y_1, y_2) we find that all of the constraints are satisfied and that the first and last are binding. Hence if $(11, 9)$ is a solution, then the corresponding solution to the primal involves $x_2 = x_3 = 0$ (because second and third dual constraints are not binding) and both primal constraints holding as equations (because both dual variables are positive). Solving the resulting system yields $x = (x_1, x_2, x_3, x_4) = (8, 0, 0, 3)$, which is feasible for the original problem. Hence the give y is a solution. There are many other ways: you can solve the original problem using the simplex algorithm and then check that y is dual feasible and yields the same value; you can graphically solve the dual.

Students who cogently and completely describe some method that will work receive 10 points and then you award 5 each for progress (in my answer, 5 points for checking feasibility, 5 points for finding a potential solution to the primal, and 5 more for verification).

6. (a) This is false. (P') has a smaller feasible set in general.
   (b) This is true. (P') is the dual of (D).
   (c) This is false: (D) is the dual of (P'), but it is not the dual of (P) (because it lacks $y \geq 0$). So values of (P) may actually be greater than values of (D).
   (d) This is true. (If you set $y = u - v$, $u, v \geq 0$, then apply the definition of dual, you will be able to obtain the equations in (D).)
   (e) This is true. (P') has a smaller feasible set than (P) in general.
   (f) This is true, for the same reason. The solution of (P') must be feasible for (P).
   (g) This is false. Left is a good strategy if Row plays (2); Center is good if Row plays (1); Right is good if Row plays (4).
   (h) This is true. The payoffs for Row's strategies are: (1): $\frac{40}{9}$; (2): $\frac{40}{9}$; (3): $\frac{17}{9}$; (4): $\frac{10}{9}$.
   (i) False. This doesn't make sense (although $p^* U q^* = v^*$).
   (j) False. U can have a negative column (so Column could guarantee a negative payoff for Row) and the row sums still be positive. For example, Row can have one strategy, which pays 10 if Column goes left and -1 if Column goes right.