Econ 172A, Fall 2003: Final Examination

Instructions.

1. The examination has six questions. Answer them all.
2. If you do not know how to interpret a question, then ask me.
3. You must justify your answers to the Questions 1 through 5. No justification is necessary for Question 6.
4. The table below indicates how points will be allocated on the exam.
5. Work alone. You may not use notes or books.
6. You have three hours.

	Score	Possible
I		60
II		60
III		25
IV		30
V		25
VI		50
Exam Total		250
Course Total		500
Grade in Course		

The first six parts of Question I are worth six points each. The last two parts are worth 12 points each. In Question II 10 points each for parts a, e, and h; 6 points for the remaining parts.

1. A company can produce two products. The table below summarizes the production technology. Each week, up to 400 units of raw material can be purchased at a cost of \$1.50 per unit. The company employs four workers, who work 40 hours per week. (Their base salaries are considered a fixed cost and do not enter the computation.) Workers are paid \$6 per hour to work overtime. Each week, 320 hours of machine time are available.

	Product 1	Product 2
Selling Price	\$15	\$8
Labor Required	.75 hour	.5 hour
Machine Time Required	1.5 hour	.8 hour
Raw Material Required	2 units	1 unit

If the firm does not advertise, 50 units of product 1 and 60 units of product 2 will be demanded each week. Advertising can be used to stimulate demand. Each dollar spent on advertising product 1 increases its demand by 10 units. Each dollar spent on advertising for product 2 increases its demand by 15 units. At most \$100 can be spent on advertising. To formulate the problem, define the following variables:

$P_1 =$ the number of units of product 1 produced each week.

$P_2 =$the number of units of product 2 produced each week.

$OT =$the number of hours of overtime labor used each week.

$RM =$number of units of raw material purchased each week.

$A1 =$amount (in dollars) spent each week advertising product 1.

$A2 =$amount (in dollars) spent each week advertising product 2.

The firm's optimization problem is then:

$$
\begin{array}{lrcrcrcrcrcrcr}
\text{max} & 15P_1 & + & 8P_2 & - & 6OT & - & 1.5RM & - & A_1 & - & A_2 & & \\
\text{subject to} & P_1 & & & & & & & - & 10A_1 & & & \leq & 50 \\
 & & & P_2 & & & & & & & - & 15A_2 & \leq & 60 \\
 & .75P_1 & + & .5P_2 & - & OT & & & & & & & \leq & 160 \\
 & 2P_1 & + & P_2 & & & - & RM & & & & & \leq & 0 \\
 & & & & & & & RM & & & & & \leq & 400 \\
 & & & & & & & & & A_1 & + & A_2 & \leq & 100 \\
 & 1.5P_1 & + & .8P_2 & & & & & & & & & \leq & 320 \\
 & & & \multicolumn{6}{l}{\text{and all variables nonnegative.}} & & & & &
\end{array}
$$

Excel's solution is on the next page. Answer as many of the questions below using the output. If there is some question that you cannot answer using the output that I have provided, explain why you cannot answer it completely and use the available information to say as much as you can. Answer each question independently of the other parts. Justify your answers.

(a) Interpret the constraint: $P_1 - 10A_1 \leq 50$. (That is, explain in words how this constraint relates to the statement of the formulation problem.)

(b) If overtime costs only \$4 per hour, would the company use it?

(c) If each unit of product 2 sold for \$8.50 would the current basis remain optimal? What would be the new solution and its value?

(d) How much would profit increase if the company obtained another unit of raw material without charge?

(e) How much would the company be willing to pay for another hour of machine time?

(f) If each worker were required (as part of the regular work week) to work 45 hours per week, what would the company's profits be?

(g) Would it be worthwhile for the firm to produce a new product that sold for \$20, could be sold in positive quantities without advertising, took 1 hour of labor per unit to produce, required 90 minutes of machine time, and 3 units of raw material per unit?

(h) An inventor offers to sell the firm a new design that would enable the firm to produce a unit of product 1 using 1.95 units of raw material instead of 2 units (the other requirements remain unchanged). The inventor is willing to rent the rights to the new design for \$30 per week. Should the firm buy the rights to the invention?

2. Here is a simplified version of baseball. Player one, the batter, can either take the pitch or swing. Player two, the pitcher, can throw either a fast ball or a curve. If player one swings at a curve or takes a fast ball, he is "out" and loses one. If player one swings at a fast ball he wins 2. If player one takes a curve ball, he walks and earns one. The game is zero sum.

   (a) Write down a payoff matrix for this game. (Label the strategies and explain what they represent.)

   (b) Does Player I have any dominated strategies? If so, identify the dominated strategies.

   (c) Does Player II have any dominated strategies? If so, identify the dominated strategies.

   (d) Find the pure-strategy security levels of both players.

   (e) Does the game have an equilibrium in pure strategies? If so, find it. If not, find the equilibrium mixed strategies and the value of the game.

   (f) Suppose that Player I knows that Player II will throw a curve 25% of the time. What should Player I do and what is his expected payoff?

   (g) Suppose that Player II knows that Player I will swing 60% of the time. What should Player II do and what is her expected payoff.

(h) Now consider a two-pitch version of the game. If the first pitch is a fast ball and the batter swings, Player I (the batter) wins 2. If the first pitch is a fast ball and the batter does not swing, the batter loses one. If the first pitch is a curve and the batter swings, the batter is out and loses one. If the first pitch is a curve and the batter does not swing, then the pitcher makes a second pitch. The payoffs for the second pitch are the same as the one-pitch version of the game that you described in part (a). Write down a payoff matrix for the two-pitch game.

3. Find the equilibrium strategies for both players and the value of the following two-player zero-sum game:

0	8
1	4
8	0

4. Write down a linear programming problem in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

with as many of the following properties as possible.

(a) A has two rows and two columns.

(b) The linear programming problem is not feasible.

(c) The dual of the linear programming problem is unbounded.

A complete answer will assign numerical values to A, b, and c and verify (you may use any method) that the resulting problem is infeasible and its dual is unbounded. If it is impossible to find a linear programming problem that satisfies all three of the conditions, explain why it is not possible and give an example that satisfies the first two properties.

5. Consider the linear programming problem:

$$\begin{array}{lrcrcrcrcl}
\text{max} & 2x_1 & - & 4x_2 & - & 6x_3 & + & 5x_4 & & \\
\text{subject to} & x_1 & + & 4x_2 & + & 8x_3 & - & 2x_4 & \leq & 2 \\
& -x_1 & + & 2x_2 & + & 4x_3 & + & 3x_4 & \leq & 1 \\
& & & & & & & x & \geq & 0
\end{array}$$

Is $(y_1, y_2) = (11, 9)$ is a solution to the dual? You may use any method to answer this question, but you must explain the method that you use and why it is appropriate. You will receive no credit for simply answering "no" or "yes."

6. For each of the statements below, circle **TRUE** if the statement is always true, circle **FALSE** otherwise. No justification is required. Please read the problem **carefully** before answering.

   For the first six problems below, (P) refers to the problem:

   $$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0,$$

   (D) refers to the problem:

   $$\min b \cdot y \text{ subject to } yA \geq c$$

   and (P') refers to the problem:

   $$\max c \cdot x \text{ subject to } Ax = b, x \geq 0$$

   (a) **TRUE** **FALSE**
   If (P) is feasible, then (P') is feasible.

   (b) **TRUE** **FALSE**
   If (D) has a solution, then (P') has a solution.

   (c) **TRUE** **FALSE**
   If x is feasible for (P) and y is feasible for (D), then $c \cdot x \leq b \cdot y$.

   (d) **TRUE** **FALSE**
   (D) is the dual of (P').

   (e) **TRUE** **FALSE**
   If (P') is feasible, then (P) is feasible.

   (f) **TRUE** **FALSE**
   If both (P) and (P') have solutions, then the value of (P') is no greater than the value of (P).

The next two problems refer to the two-player zero-sum game below.

	LEFT	CENTER	RIGHT
1	8	4	10
2	0	5	12
3	9	1	50
4	10	0	-50

(g) **TRUE** **FALSE**
Column has a dominated strategy.

(h) **TRUE** **FALSE**
If Column plays Left with probability $\frac{1}{9}$, Center with probability $\frac{8}{9}$, and Right with probability 0, then the expected payoff to Row can be no greater than $\frac{40}{9}$.

For the next two questions, let U be the payoff matrix of a two-player zero sum game (so that the entry in row i and column j of U is the payoff to the row player if he plays his ith strategy and the column player plays her jth strategy). Let p^* be the row player's equilibrium mixed strategy, let q^* be the column player's equilibrium mixed strategy, and let v^* be the value of the game.

(i) **TRUE** **FALSE**
$q^*Up^* = v^*$.

(j) **TRUE** **FALSE**
If all of the rows of U have positive sums, then $v^* > 0$.