

Allocation to Industry Portfolios under Markov Switching Returns

Deniz KEBABCI

August 29 , 2005

Abstract

This paper proposes a Gibbs Sampling approach to modeling returns on industry portfolios. We examine how parameter uncertainty in the returns process with regime shifts affects the optimal portfolio choice in the long run for a static buy-and-hold investor. We find that after we incorporate parameter uncertainty and take into account the possible regime shifts in the returns process, the allocation to stocks can be smaller in the long run. We find this result to be true for both the NASDAQ portfolio and the individual high tech and manufacturing sector portfolios. Finally, we include dividend yields and the T_ bill rate as predictor variables in our model with regime switching returns and find that the effect of these predictor variables is minimal: the allocation to stocks is still generally smaller in the long run.

1 Introduction

Since Kandel and Stambaugh (1996), how predictability in asset returns affects optimal portfolio choice has been a widely asked question. In this context, particular attention has been paid to estimation risk, in other words, to the uncertainty about the true values of model parameters. We show in this paper that after incorporating parameter uncertainty, investors generally allocate less to stocks the longer the horizon. To this end, we focus on the effect of regime switching behavior in returns on the optimal portfolio choice, with and without parameter uncertainty and with and without extra predictor variables. The effect of regime switching behavior in returns on the optimal portfolio choice

with parameter uncertainty, to the best of our knowledge, has not been studied in the literature before.

We focus on the predictability of returns and asset allocation at the industry level. Our paper is different from papers in the literature in this respect as well because previous research uses market indices to study the effect of predictability in returns on asset allocation with parameter uncertainty and does not pay much attention neither to the predictability of returns at the industry level nor the implications of this on asset allocation, with and without parameter uncertainty.

Our paper is a continuation of the work that started with Samuelson (1969) and Merton (1969), where they show that if asset returns are i.i.d., an investor with power utility who rebalances his portfolio optimally should choose the same asset allocation, regardless of investment horizon. Barberis (2000) concludes that in light of the growing body of evidence that returns are predictable, the investor's horizon may no longer be irrelevant. Our paper is closest, in this respect, to Barberis (2000) since we also look at the sensitivity of optimal asset allocation to the investor's horizon.

However, our paper is different from Barberis (2000) in that we claim here that even if returns are not predictable by a predictor variable, the regime switching behavior in returns also makes the investor's horizon relevant to the portfolio decision. Barberis (2000)'s main focus, on the other hand, is on the comparison between the cases with i.i.d. returns vs. predictability in asset returns by the dividend yield and on how they affect the optimal portfolio choice, with and without parameter uncertainty.

Barberis (2000)'s paper, in this respect, is closest to Kandel and Stambaugh (1996), since Kandel and Stambaugh (1996) also point towards the importance of recognizing parameter uncertainty and use a Bayesian setting in the context of asset allocation with predictable returns, but they do not look at the horizon effects: they only consider one period ahead predictions.

Our results with utilizing a regime switching model for returns and analyzing the effect of this specification on optimal portfolio choice, are in line with Bodie (1995) and Samuelson (1994) in the debate between whether investors with long horizons should allocate more heavily to stocks or not, against Siegel (1994) which claims that they should. Samuelson (1994) points out the widely held fallacies and misconceptions around this debate.

Our results also agree with Guidolin and Timmermann (2004) where they employ a four-state Markov switching model for returns and look at the asset allocation implications of this specification. Guidolin and Timmermann (2004) do not take into account the effect of parameter uncertainty in modeling returns using Bayesian algorithms, but they update their parameter estimates using an EM algorithm.

A broader range of papers that study the issue of parameter uncertainty include Bawa, Brown, and Klein (1979), Jobson and Korkie (1980), Jorion (1985), Frost and Savarino (1986). Bawa, Brown, and Klein (1979) focuses more on how estimation risk varies with size of data sample, while keeping investor's horizon fixed. We also study in this paper the effect of the size of data sample with a fixed horizon and find out that the size of data sample matters.

Another widely asked question in the finance literature is whether we can distinguish distinct regimes in stock market returns. Some of the previous papers that have used the techniques proposed by Hamilton (1989) include Schwert (1989), Turner, Startz, and Nelson (1989), Hamilton and Susmel (1994), Van Norden and Schaller (1993)¹. Schwert (1989) considers a model in which returns may have either a high or low variance and switches between these return distributions following a two-state Markov process. Hamilton and Susmel (1993) propose a model with sudden discrete changes in the process which governs volatility. Turner, Startz, and Nelson (1989) consider a Markov switching model in which either the mean, the variance, or both may differ between two regimes. Finally, Van Norden and Schaller (1993) allow the probability of transitions from one regime to another depend on economic variables and they find very strong evidence of switching behavior. Van Norden and Schaller (1993) also ask whether returns are predictable, even after accounting for regime switches.

Regime switching models capture many of the properties of asset returns that emerge from the empirical studies such as having regimes with very different mean, volatility and correlations across assets. They are good in capturing fat tails and skews in the distribution of asset returns as well as identifying time-varying expected returns, volatility persistence and asymmetric correlations due to the underlying state probabilities. As is put in Guidolin and Timmermann (2002), they also serve as accommodators of outliers in multi-state models, for instance having a crash state capturing large negative returns and a bull burst state capturing large positive returns.

In light of these papers, we consider a two-state Markov switching mean-variance model close in spirit to Kim and Nelson (1998). We use a Gibbs Sampling method to account for the uncertainty about the parameters of the Markov switching process, namely the mean, variance and the transition probabilities.

Allowing the returns to have different means and variances in different states has strong implications for asset allocation. For instance, if stock market volatility is higher in recessions than in expansions, equity investments are less attractive in recessions (as long as their mean returns do not rise substantially). Also, for instance, knowing that the current state is a persistent bull market will make equities more attractive. Our paper is close in spirit to Pettenuzzo and Timmermann (2004) and Guidolin and Timmermann (2002) in this context.

¹Ang and Bekaert (2002), Guidolin and Timmermann (2004), Perez-Quiros and Timmermann (2000) are some other papers that use Markov switching models.

Pettenuzzo and Timmermann (2004) suggest that there are structural breaks in the parameters of the return prediction model and this might affect the asset allocation problem. But we claim that there is no firm reason to believe that these are indeed structural breaks and not recurring regimes. In other words, we'd like to see in this paper if the "history repeats." And our paper is different than Guidolin and Timmermann (2002), which also studies strategic asset allocation with regime switching in asset returns, since we use a Bayesian analysis to take into account the parameter uncertainty in the returns process and a different set of risky assets.

We also include, in this paper, the optimal allocation results with the linear case to be comparable to our original model with regime switching returns, and two extra predictor variables besides the Markov switching process to check for extra predictive power of these two variables.

Finally, but not least, the not so many papers on industry stock return predictability and asset allocation include Cavaglia and Moroz (2002), Sorenson and Burke (1986), Beller, Kling and Levinson (1998), Capaul (1999), Fama and French (1988a), Ferson and Harvey (1991), Lo and MacKinlay (1996), and Moskowitz and Grinblatt (1999). Cavaglia and Moroz (2002) focus on a cross-industry, cross-country allocation framework for making active global equity investment decisions. Sorenson and Burke (1986) find that US industry returns can be predicted by using past return performance. Capaul (1999) focuses on the predictive ability of different factors (such as Fama and French (1988) factors: value, size and momentum, etc.) on global industries. Fama and French (1988a) estimate AR models for returns of portfolios based on industry classifications. Ferson and Harvey (1991) and Lo and MacKinlay (1996) both investigate industry groups together with size deciles and bond portfolios. Moskowitz and Grinblatt (1999) attribute the momentum in intermediate term individual stock returns to industry momentum. Beller, Kling and Levinson (1998) investigate in-sample and out-of-sample predictability of excess returns over the period 1973-1996. This paper is the closest in spirit to ours since it predicts returns with a Bayesian multivariate regression model. Some of the predictor variables they use are: term spread, default spread, aggregate dividend yield, etc.

Most of the papers mentioned in the previous paragraph do not consider the optimal asset allocation implications of the predictability of industry stock returns.

The reason why we think the industry asset allocation problem is interesting is because it might be a more advantageous manner to engage in asset allocation. The low correlations across industries can be made use of in diversification practices. In the age of globalization, and region-specific developments, such as the introduction of Euro, etc., one might put a light on the question of whether there are additional gains from industry asset allocation compared to the widely practised international asset allocation, with the help of our paper. One of the

reasons why we think that industry asset allocation hasn't been widely embraced by the practitioners so far is because the predictability of industry returns is thought to be low. We think that our paper contributes to the academics literature in this respect.

We first do the analysis with the NASDAQ portfolio, because we think that this might be a good comparison with the previous studies done in the literature with different market portfolios (e.g., NYSE, etc.). We also think that we can compare the NASDAQ portfolio results with the high tech sector results to show the similarities or differences between the results with the two datasets which are generally thought to be overlapping. We then consider two sector portfolios, high tech and manufacturing portfolios, as the basis for our analysis.

We finally look at the optimal asset allocation decision with two risky assets, high tech and manufacturing portfolios, versus the risk free asset and compare this with the optimal asset allocation decision with one risky asset, either high tech or manufacturing portfolio, versus the risk free asset. Our paper, to the best of our knowledge, is also a first in studying the optimal asset allocation decision with two risky assets versus the risk free asset in a framework that incorporates parameter uncertainty.

2 Data

Data used in this paper includes continuously compounded returns on the value-weighted portfolio of NASDAQ for months between 1973/01 and 2003/12 and 1 month Treasury bill rates, and the value-weighted portfolios of high tech and manufacturing sectors for months between 1926/11 and 2003/12. All data is obtained from CRSP database. The 30 day Treasury bill rates are given under Treasury and Inflation Indices (1000708-T30IND), and the monthly NASDAQ value-weighted market index returns are given by the code 100060.

Excess returns are calculated by subtracting the continuously compounded T_bill rate from the continuously compounded returns of each series. We calculate the continuously compounded returns for the high tech and manufacturing sectors with the following formula: $R_{t+1}^{h,m} = \ln P_{t+1}^{h,m} - \ln P_t^{h,m}$, where $R_{t+1}^{h,m}$ refers to the returns for high tech and manufacturing sectors respectively, and $P_{t+1}^{h,m}$, and $P_t^{h,m}$ refer to the one period ahead and current value-weighted prices for high tech and manufacturing sectors respectively.

We create the value-weighted sector prices used in the calculation of sector returns by multiplying the price of each individual stock in the sector with its lagged weight in the sector and summing these products up. We obtain the primary industry classifications from K.R. French's website². The value-weighted

²<http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>

portfolios of the two sectors cover the whole NYSE, AMEX, and NASDAQ stocks given in the CRSP database that fall under the four digit SIC codes given on K.R. French's website.

We also report the results for data after 1953/01 for both high tech and manufacturing sectors. We check the results with this restricted postwar data because we are interested in seeing whether the fact that interest rates were held almost constant by Federal Reserve before the Treasury Accord of 1951 affects our analysis.

The descriptive statistics on returns are given in figures 1 and 2 at the end of the paper. These tables show among many things that as sample size gets smaller, the mean returns increase for the Treasury bill and manufacturing sector and decrease for the high tech sector. The standard deviations decrease for the Treasury bill and manufacturing sector and increase for the high tech sector, with sample size. Kurtosis is largest for high tech, and skewness is negative for high tech and manufacturing sectors, and positive for Treasury bill. The minimum is the largest for high tech sector.

These figures also show that high tech sector has the lowest mean returns among all except for the longest dataset (1926/12-2003/12). In the period 1973/01-2003/12, NASDAQ has the highest mean returns. High tech sector also has the highest standard deviation among all in all time periods.

We also give the descriptive statistics on dividend yields for both the high tech and manufacturing sectors in figure 3. The dividends are also obtained from the CRSP database. The dividend yields for each sector is calculated by multiplying the dividend yield for each individual stock within a sector with its lagged weight in that sector.

3 Methodology

We analyze asset allocation in discrete time for an investor with CRRA utility over terminal wealth. We consider two assets: Treasury bills and a stock index. The investor uses a Markov switching model to forecast returns. We incorporate the parameter uncertainty taking a Bayesian approach, where the uncertainty about the parameters is summarized by the posterior distribution of the parameters. We then will compare the case with uncertainty with the case without uncertainty where the distribution of returns is constructed conditional on fixed parameter estimates.

3.1 Asset Allocation Framework for a Buy-and-Hold Investor

Suppose we are writing down the portfolio problem for a buy-and-hold investor with a horizon of \widehat{T} months and we're initially at time T . Suppose further that we have two assets: Treasury bills and a stock index. We suppose that the continuously compounded monthly return on Treasury bills is a constant r_f . In our numerical framework we set r_f equal to 0.000361, which is the return on December 2003 (the last month of our sample) of the one month Treasury bill (since this return is unusually small, we also try the average return on one month T-bill rates over our samples). We model the excess returns on the stock index using a Markov switching framework. It takes the form

$$r_t = \mu_{s_t} + e_t, \quad e_t \sim N(0, \sigma_{s_t}^2), \quad (1)$$

$$\mu_{s_t} = \mu_1 S_{1t} + \mu_2 S_{2t}, \quad (2)$$

$$\sigma_{s_t}^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t}, \quad (3)$$

$$S_{jt} = 1, \text{ if } S_t = j, \text{ and } S_{jt} = 0, \text{ otherwise, } j = 1, 2, \quad (4)$$

$$p_{ij} = \Pr[S_t = j | S_{t-1} = i]. \quad (5)$$

If initial wealth $W_T = 1$ and w is the allocation to the stock index, then end-of-horizon wealth is given by

$$W_{T+\widehat{T}} = (1-w) \exp(r_f \widehat{T}) + w \exp(r_f \widehat{T} + r_{T+1} + r_{T+2} + \dots + r_{T+\widehat{T}}). \quad (6)$$

We ignore intermediate consumption (the investor is assumed to consume end of period wealth, $W_{T+\widehat{T}}$).

Then, the investor's preferences over terminal wealth are given by a CRRA utility function, $u(W) = \frac{W^{1-A}}{1-A}$, and the buy-and-hold investor's problem is to solve

$$\max_w E_T \left(\frac{\{(1-w)\exp(r_f \widehat{T}) + w \exp(r_f \widehat{T} + r_{T+1} + r_{T+2} + \dots + r_{T+\widehat{T}})\}^{1-A}}{1-A} \right). \quad (7)$$

We then consider two cases to calculate this expectation, one without parameter uncertainty and one with parameter uncertainty. In the case without parameter uncertainty the investor solves

$$\max_w \int u(W_{T+\widehat{T}}) p(R_{T+\widehat{T}}|y, \widehat{\theta}) dR_{T+\widehat{T}}, \quad (8)$$

where y is the data observed by the investor up until the start of his investment horizon ($y=(r_1, r_2, r_3, \dots, r_T)'$) and $R_{T+\widehat{T}} = r_{T+1} + r_{T+2} + \dots + r_{T+\widehat{T}}$ is the cumulative return. $p(R_{T+\widehat{T}}|y, \widehat{\theta})$ is the distribution for future stock returns conditional on a set of parameter estimates, $\widehat{\theta}$, and y . In the case with parameter uncertainty the problem becomes

$$\max_w \int u(W_{T+\widehat{T}}) p(R_{T+\widehat{T}}|y, \theta) p(\theta|y) dR_{T+\widehat{T}} d\theta, \quad (9)$$

where $p(\theta|y)$ is the posterior distribution and $p(R_{T+\widehat{T}}|y, \theta)$ is the likelihood. In this paper, we consider a normal likelihood. In words, the last maximization problem means that, rather than constructing the distribution of future returns conditional on fixed parameter estimates, we integrate over the uncertainty in the parameters captured by the posterior distribution.

We approximate the integral for expected utility by taking a sample $(R_{T+\widehat{T}}^{(i)})_{i=1}^I$ from one of the two distributions (distribution of returns conditional on fixed parameter values or predictive distribution obtained by using the posterior distributions and the likelihood) and then for $w=0, 0.01, 0.02, \dots, 0.99$ computing

$$\frac{1}{I} \sum_{i=1}^I \frac{\{(1-w)\exp(r_f \widehat{T}) + w \exp(R_{T+\widehat{T}}^{(i)})\}^{1-A}}{1-A}. \quad (10)$$

We then report the value of w that maximizes the above expression.

In the case without parameter uncertainty, we take 10,000 independent draws (indeed 12,000 draws, but we discard the first 2000 draws) from the normal distribution with mean and variance equal to the posterior mean and posterior

variance. In appendix A we explain in detail how we find the posterior distributions. For the case with parameter uncertainty, we use the posterior distribution to obtain the predictive distribution. After we find the posterior distributions, sampling from the predictive distribution is equivalent to first sampling from the posterior distributions and then the likelihood. This is to say that for each of the 10,000 (μ, σ^2) pairs drawn, we sample once from the normal distribution³.

3.2 Markov Switching Models and Gibbs Sampling

The basic difference of the Gibbs Sampling approach to inference on Markov switching models from the classical approach is that in the Bayesian analysis, both the parameters of the model and the Markov switching variable, S_t , $t = 1, 2, \dots, T$ (one doesn't observe S_t , just knows that it is an outcome of an unobserved, discrete-time, discrete-state Markov process), are treated as random variables. Therefore, in contrast to the classical approach, inference on \widetilde{S}_T ($= [S_1 S_2 \dots S_T]'$) is based on a joint distribution. In the classical approach, inference on Markov switching models consists of first estimating the model's unknown parameters, then making inferences on the unobserved Markov switching variable, S_T , conditional on the parameter estimates. In Bayesian approach, both the parameters of the model and the unobserved Markov switching variables are treated as missing data, and they are generated from appropriate conditional distributions using Gibbs Sampling.

3.3 Gibbs Sampling

The main model we are using in this paper is the Bayesian alternative to the analysis of the two-state Markov switching mean-variance model of returns (equations 1-5).

The Gibbs Sampling procedure is given by successive iteration of the following steps:

- Step 1 Generate \widetilde{S}_T , conditional on $\widetilde{\sigma}^2, \widetilde{\mu}, \widetilde{p}, \widetilde{r}_T$
- Step 2 Generate \widetilde{p} , conditional on \widetilde{S}_T
- Step 3 Generate $\widetilde{\sigma}^2$, conditional on $\widetilde{\mu}, \widetilde{r}_T, \widetilde{S}_T$
- Step 4 Generate $\widetilde{\mu}$, conditional on $\widetilde{\sigma}^2, \widetilde{r}_T, \widetilde{S}_T$

³We should note at this point that the reason why we don't use quadrature methods as in Ang and Bekaert (2002), instead of the simulations employed here, to evaluate the integrals is because the integral in the case without parameter uncertainty is not one-dimensional. Quadrature methods are a good alternative when the integral is one-dimensional like in the 'without parameter uncertainty' case or in the special case when $\widehat{T} = 1$. Kandel and Stambaugh (1996), for instance, use quadrature methods; since, then, using a $\widehat{T} = 1$, they can obtain a closed-form solution for the predictive distribution.

In the case with predictor variables, we include an additional step, Step 5, where we generate $\tilde{\phi}$, conditional on $\tilde{\sigma}^2, \tilde{\mu}, \tilde{S}_T$, where $\tilde{\phi}$ represents the coefficients on the predictor variables.⁴

4 Asset Allocation with Different Models

4.1 Empirical Results with the Linear Model

Figure 4 gives the parameter estimates (posterior means) for the linear model (no Markov switching returns and no predictor variables). More explicitly, the model that we base our results on, in figure 4, is:

$$r_t = \mu + \varepsilon_t, \tag{11}$$

where r_t is the continuously compounded excess return in month t , and $\varepsilon_t \sim N(0, \sigma^2)$. We run this regression for each of the three series (NASDAQ, high tech, manufacturing). One can see from these estimates that high tech sector has a lower posterior mean return and a higher posterior standard deviation than manufacturing sector with both the shorter and longer datasets. NASDAQ, although not directly comparable since its estimates are calculated with a different time period, has a higher posterior mean return and a higher posterior standard deviation than both sectors across all time periods.

We give the optimal allocations w for the three different series (NASDAQ, high tech, manufacturing) versus the risk free rate, for a variety of risk aversion levels A , and with and without parameter uncertainty for the linear model in figure 10. We give the results for a 10 year investment horizon and utilizing the longer dataset we have (where the time period is 1926/12-2003/12) in figure 10. The vertical axis in this figure represents the optimal weights and the horizontal axis represents the horizon in months. The results look similar to the results in Barberis (2000). The optimal allocation to high tech sector stocks is lower than the optimal allocation to NASDAQ and manufacturing sector stocks, and all series exhibit almost flat optimal allocation levels throughout the investment horizon of 10 years. These results also confirm the Samuelson (1969) and Merton (1969) results for a buy-and-hold investor⁵.

⁴Generations of these conditional distributions are explained in further detail in Appendix A, a la Kim and Nelson (1998). Please refer to Kim and Nelson (1998) for more details on the Gibbs Sampling method.

⁵Note that Samuelson (1969) and Merton (1969) analyze the setting with i.i.d. returns and optimal rebalancing, where we analyze the setting with a linear model and a buy-and-hold investor.

4.2 Empirical Results with the Regime Switching Model

In this section, we run the univariate regressions of returns on the NASDAQ, high tech and manufacturing indices, and find the optimal portfolio weights w when these returns are used in the optimal asset allocation problem versus risk free rates.

The model for high tech and manufacturing sectors is as follows⁶:

$$r_t^h = \mu_{s_t}^h + \varepsilon_t^h \quad (12)$$

$$r_t^m = \mu_{s_t}^m + \varepsilon_t^m \quad (13)$$

In this model, there's not an impact of manufacturing sector on high tech sector and vice versa. We assume that sectors have 'independent' processes in this model. We run the equations separately, calculate the predictive distribution of returns from the posterior distributions (or posterior means for the case without parameter uncertainty) obtained from each run of the Gibbs Sampling procedure and obtain the optimal asset allocation of each index versus the risk free asset.

This model follows a similar structure to the model we introduced in equations 1-5, where both the means and variances of the process are regime dependent. In the results given in this section, sig_1 refers to the variance in the first state, and sig_2 refers to the variance in the second state; whereas mu_1 refers to the mean in the first state, and mu_2 refers to the mean in the second state.

Figures 5-6 report the parameter estimates following a Bayesian Gibbs Sampling approach to a two-state Markov switching mean-variance model of excess returns. Monthly CRSP data for the period 1973/01-2003/12 is utilized for the NASDAQ portfolio and monthly CRSP data for the period 1926/11-2003/12 is utilized for the high tech and manufacturing sector portfolios in Figure 5; and monthly CRSP data for the period 1953/01-2003/12 is utilized for the high tech and manufacturing sector portfolios in Figure 6.

As we can see from figure 5, the persistence levels of states for the NASDAQ portfolio is about 9 and 29 months respectively for states 1 and 2. For manufacturing and high tech sectors, the persistence levels are about 9, 56 and 6, 15 for the first and second states with the long dataset. The first state for NASDAQ is the state with the higher variance and lower mean excess return

⁶We also tried to include different order AR processes in our model, but found no significant effect of lags on the process, so we decided to leave the lags out.

(indeed negative), which is also the state in which NASDAQ stocks stay less. (The allure of the NASDAQ stocks might be this, that the investors even if they assume that they're in the first state, knowing that it's not going to last long might be overinvesting.) With the long dataset, the manufacturing and high tech sectors also stay less in the first state and the first state is the state with lower mean excess returns (negative) and higher variances. In comparing (if at all) the NASDAQ index with the high tech sector, one should keep in mind the different time periods utilized for the two series.

The persistence levels are 4, 33 and 6, 46 for states 1 and 2, for manufacturing and high tech sectors, respectively, with the short dataset. With the short dataset, again, for both the manufacturing and high tech sectors, state 1 is the state with negative mean excess returns and higher variances, and where both series stay less.

The posterior distributions given in figures 8 and 9 are calculated by generating 12,000 draws. The first 2,000 draws are discarded. These figures give the posterior probabilities of the two different states for the three series we use in our paper: NASDAQ portfolio, high tech and manufacturing sector portfolios with the long and short datasets respectively. It seems like, roughly, the regimes shift around 1929, 1938, 1942, 1960, 1974, 1982, 1996 for the high tech sector; around 1928, 1937, 1974, 1980, 1987, 1999 for the manufacturing sector, with the longer dataset; and around 1973, 1978, 1980, 1987, 1990, 1998 for NASDAQ. With the shorter dataset, for the high tech sector, the regime shifts seem like taking place, roughly, around 1958, 1960, 1984, 1987, 1996; and with the shorter dataset, for the manufacturing sector, the regime shifts seem like taking place, roughly, around 1955, 1974, 1980, 1987, 1999.

We present the optimal allocations w which maximize the expected utility for a variety of risk aversion levels A , with an investment horizon \bar{T} of 10 years and for different cases where the investor either ignores or accounts for parameter uncertainty with the two-state Markov switching model in figures 11-13. In figures 11-13, as in figure 10, the vertical axis represents the optimal weights and the horizontal axis represents the horizon in months. Figures 11-12 give the case with parameter uncertainty and figure 13 gives the case without parameter uncertainty. Figure 11 gives the results for the longer sample period (1926/12-2003/12) for the manufacturing and high tech sectors, and figure 12 gives the results for the shorter sample period (1953/01-2003/12) for the manufacturing and high tech sectors. Figure 11 also includes the results for NASDAQ for the sample period 1973/01-2003/12. For the case without parameter uncertainty, we do not give the results for the shorter dataset, since these results look similar to the results with the longer dataset.

We find out from these figures that the static buy-and-hold investor allocates less to equities as the horizon increases for the NASDAQ and high tech series once parameter uncertainty is taken into account. The reason why the

investor allocates less to equities as the horizon increases is because incorporating uncertainty increases the variance of the distribution for cumulative returns, particularly at longer horizons. This makes stocks look riskier to a long-term buy-and-hold investor reducing their attractiveness.

The effect is smaller for manufacturing with the long dataset (figure 11), indeed optimal allocation to manufacturing stays around the same level throughout the whole investment horizon for the different risk aversion levels. The effect for manufacturing actually reverses in the short dataset (figure 12): the static buy-and-hold investor allocates more to equities as the horizon increases. This behavior might be due to the state the manufacturing sector is perceived to be in at the time of the forecast.

The more risk-averse the investor is (the higher the A is), the smaller the optimal allocation to stocks is in all three series, and all figures. As can be seen in figure 11, when the parameter uncertainty is taken into account, the investor allocates a big portion of his wealth to stocks only if he's very risk loving for NASDAQ. Otherwise, the optimal allocation decreases in less than a year. This result is also true for high tech: the optimal allocation with parameter uncertainty for all risk aversion levels decreases in less than a year for high tech.

One last point to note is that, in general, the optimal allocation to NASDAQ versus the risk free asset is much higher than the optimal allocation to the individual sectors versus the risk free asset, but one should be cautious with taking this result any further since the results for the individual sectors and the results for NASDAQ are based on different time periods, as we mentioned earlier.

On the other hand, when the parameter uncertainty is not taken into account (figure 13), the allocation to stocks is bigger than the allocation to stocks with parameter uncertainty being taken into account (figure 11). This result is true for all our series and for all the different sample periods and risk aversion levels we utilize, which is expected, since taking into account the parameter uncertainty always makes the stocks look riskier.

In figures 11-13, when we utilize the sample averages of the risk free rate instead of the time T (last period) risk free rate, we find similar results, so we do not see the need to exhibit those findings.

4.3 Empirical Results with Predictor Variables

In this section, we give the results for the two-state Markov switching model when we include the dividend yield and T-bill rate as extra predictor variables in the model. Figure 7 gives the parameter estimates for this model. It shows that

the coefficient for the T-bill rate is negative (ϕ_1) and the coefficient for the dividend yield is positive (ϕ_2) as one would expect. State 2 is still the state with higher mean excess returns and lower variances, although the mean excess returns are negative for both states for both sectors now. The variances remain lower for the second state, and higher for the first state. The persistence levels are about the same as they are before the predictor variables are included in the model: approximately 9 and 50 months for the manufacturing sector and 7 and 18 months for the high tech sector, for the first and second states respectively.

The graphs for this model with parameter uncertainty are given in figure 14. The results without parameter uncertainty are similar with weights slightly higher than the weights with parameter uncertainty, so we do not see the need to exhibit those results. We only give the results for the longer dataset (1926/12-2003/11 sample period). We utilize different values for the dividend yield and T-bill rate in figure 14 to show how the prediction changes when we change these values. The graph at the top in figure 14 utilizes the sample mean of the dividend yield for high tech and T-bill rate, which are given in figures 1 and 3. The graph in the middle gives the results with utilizing the sample maximum of the dividend yield for high tech and T-bill rate, which are also given in figures 1 and 3. We do not give the results with utilizing the last period (time T) dividend yield for high tech and T-bill rate since they are unusually small. The graph at the bottom gives the results with utilizing the sample maximum of the dividend yield for manufacturing and T-bill rate. We do not give the results with utilizing the last period value and sample mean of the dividend yield for manufacturing and T-bill rate since they are comparatively small, and the optimal allocation to stocks with these values looks really low throughout the whole investment horizon.

We can see from figure 14 that including dividend yield and T-bill rate as extra predictor variables in the two-state Markov switching model, depending on the initial values of dividend yield and T-bill rate employed, changes the magnitude of the allocation to stocks versus the risk free asset, but it doesn't reduce the riskiness of stocks as the horizon increases. Indeed, the optimal allocation to stocks still decreases as the horizon increases. This in our opinion shows that the predictive power of the dividend yield and T-bill rate at long horizons is not strong enough to overcome the impact of uncertainty.

4.4 Empirical Results with Two Risky Assets versus the Risk Free Asset

We also calculate the optimal asset allocation of two risky sector indices (high tech and manufacturing portfolios) versus the risk free asset to show the sector-wise optimal asset allocation decision implications of Markov switching returns

with parameter uncertainty, hence the reason why we do our analysis on sector indices.

In this case, we approximate the integral for expected utility by taking a sample $(R_{T+\hat{T}}^{h(i)})_{i=1}^{i=I}$ from the predictive distribution of the high tech excess returns and another sample $(R_{T+\hat{T}}^{m(i)})_{i=1}^{i=I}$ from the predictive distribution of the manufacturing excess returns, which we calculate using the likelihood and posterior distributions of returns using the model in section 4.2 (equations 12-13). We then for $w^h=0,0.01,0.02,\dots,0.99$ and $w^m=0,0.01,\dots,0.99$ (with the condition that $w^h+w^m<1$) compute:

$$\frac{1}{I} \sum_{i=1}^I \frac{\{(1 - w^h - w^m) \exp(r_f \hat{T}) + w^h \exp(R_{T+\hat{T}}^{h(i)}) + w^m \exp(R_{T+\hat{T}}^{m(i)})\}^{1-A}}{1 - A}. \quad (14)$$

We report the (w^h, w^m) pairs that maximize the above expression in figures 15 and 16. Figure 15 is with a ten year horizon, with the longer dataset (1926/12-2003/12), for risk aversion levels of 2 and 5, and figure 16 is with a ten year horizon, with the longer dataset, for risk aversion levels of 10, 20 and 100.

We conclude from figures 15 and 16 that, as would be expected, the optimal allocation to risk free asset increases with risk aversion level and with time. We also conclude that the optimal asset allocation to high tech stocks in the presence of manufacturing stocks (besides the risk free asset) decreases more severely than the case without the manufacturing stocks being present. The allocation to manufacturing stocks with low risk aversion levels ($A=2$ for instance) actually increases as the horizon increases. The optimal allocation to stocks (either sector) is minimal with very high risk aversion levels ($A=10,20,100$ for instance).

4.5 Estimation with a Multivariate (Two Sector) Model

An extension of the analysis to include a regime switching vector auto regression (RSVAR) to capture a variety of interactions between the two sectors will be as follows:

$$\begin{bmatrix} r_t^h \\ r_t^m \end{bmatrix} = \begin{bmatrix} \mu_{s_t}^h \\ \mu_{s_t}^m \end{bmatrix} + \begin{bmatrix} \phi_1^h \dots \phi_3^h \\ \phi_3^m \dots \phi_1^m \end{bmatrix} \begin{bmatrix} r_{t-1}^h \\ r_{t-1}^m \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^m \end{bmatrix} \quad (15)$$

where we assume $\varepsilon_t^h \sim N(0, \sigma_{s_t}^{2^h})$ and $\varepsilon_t^m \sim N(0, \sigma_{s_t}^{2^m})$.

This model focuses on two sectors, and follows an AR(1) process, but since we doubt that the AR coefficients will be significant, the second right hand side term can be ignored in the estimation practices after running initial regressions. In this model, both the means and variances are still regime dependent, and the covariance matrices reveal the correlations between the two sector returns.

One can estimate the model with and without assuming parameter uncertainty, with and without assuming perfect correlation across states in the two sectors. In the case of only two sectors, there's not much necessity to assume perfect correlation (since the number of total states in the RSVAR model, if each sector return follows a two-state regime switching model in the univariate regressions, will only be four, 2^2), but if we decide to include more than two sectors (e.g., n), since we will have to estimate a 2^n transition probability matrix, and this is hardly tractable, it'll be quite convenient to assume perfect correlation across the states of the n sectors, although this will put a doubt on the reliability of the estimation results.

4.6 More Robustness Checks

Besides using different sample periods, risk aversion levels, and series; we also employed different initial values and priors for the series we investigate and ran different number of simulations to see if 10,000 draws was giving us accurate enough answers or not, and we saw that changing the initial values, priors or the number of simulations didn't change our results significantly.

5 Conclusion

Some investors might ignore that returns follow a Markov switching process, others might instead ignore the uncertainty regarding the parameters of this Markov switching process. We show in this paper that ignoring the parameter uncertainty leads the investor overallocate to stocks, when returns follow a Markov switching process.

We also find in this paper that the static buy-and-hold investor allocates less to equities as the horizon increases for two (NASDAQ, high tech) out of the three series we analyze (NASDAQ, high tech, manufacturing) once the nonlinearity in the returns process is incorporated. For the shorter dataset we employ (using postwar data), the optimal allocation to manufacturing sector versus the risk free asset actually increases as the horizon increases. This brings us to the conclusion that some investment advisors' suggestion that long horizon investors should allocate more aggressively to equities maybe too hasty and one should also

consider the type of stocks one invests in and take into account parameter uncertainty and the possible regime switches as well. Indeed taking into account the observation that different series behave differently as the horizon increases might actually help with the portfolio diversification decisions. We also should note that this result, that is contrary to Barberis (2000), which says that one should allocate less to equities with time, is not unique in the literature. Massimo and Timmermann (2005) and Pettenuzzo and Timmermann (2004) also find similar results but as we noted earlier, in different scenarios, and they do not consider the behavior of different series (sectors) and the impact this might have on optimal stock allocation.

In this paper, we also look at the additional predictive power of the dividend yield and T-bill rate in the presence of a two-state Markov switching model, and conclude that the predictive power of these two predictor variables is not strong enough to adverse the effect of parameter uncertainty in the long run, unlike what Barberis (2000) suggests.

We also believe that results to an optimal asset allocation problem, as Bawa, Brown, and Klein (1979) suggest, are sensitive to the sample size used in the estimations, since parameter estimation uncertainty is a bigger problem with shorter datasets.

We also think that the uncertainty underlying the current state any series is in is crucial to the optimal asset allocation decision (that's why we use Gibbs Sampling methods, and take the current state as unknown). We believe that, incorrectly assuming the current state to be in a certain state (either state 1 or 2) would have significant costs, and taking into account the uncertainty about the true state of the series actually reduces these costs.

6 Appendix A

6.1 Generating \widetilde{S}_T , conditional on $\widetilde{\sigma}^2, \widetilde{\mu}, \widetilde{p}, \widetilde{\phi}, \widetilde{r}_T$

This refers to simulating $S_t, t = 1, 2, \dots, T$, as a block from the following joint conditional distribution: $g(\widetilde{S}_T | \widetilde{\sigma}^2, \widetilde{\mu}, \widetilde{p}, \widetilde{\phi}, \widetilde{r}_T)$. To do this, first consider the derivation of the joint conditional density: $g(\widetilde{S}_T | \widetilde{r}_T)$. It's easy to show that $g(\widetilde{S}_T | \widetilde{r}_T) = g(S_T | \widetilde{r}_T) \prod_{t=1}^{T-1} g(S_t | S_{t+1}, \widetilde{r}_T)$, using the Markov property of S_t that conditional on S_{t+1} , for example S_{t+2}, \dots, S_T , and r_{t+1}, \dots, r_T contain no information beyond that in S_{t+1} . To derive the terms in the last equality, the following steps can be employed. First, run Hamilton's (1989) basic filter to get $g(S_t | \widetilde{r}_T)$, $t = 1, 2, \dots, T$, and save them. The last iteration would provide us $g(S_T | \widetilde{r}_T)$. In the second step to generate $g(S_t | S_{t+1}, \widetilde{r}_T)$, for $t = 1, \dots, T-2, T-1$, we make use of the following result: $g(S_t | S_{t+1}, \widetilde{r}_T) \propto g(S_{t+1} | S_t) g(S_t | \widetilde{r}_T)$. Using this last equation generating S_t is the same as first calculating $\Pr[S_t = 1 | S_{t+1}, \widetilde{r}_T] = \frac{g(S_{t+1} | S_t=1) g(S_t=1 | \widetilde{r}_T)}{\sum_{j=1}^2 g(S_{t+1} | S_t=j) g(S_t=j | \widetilde{r}_T)}$, then generating a random number from the uniform distribution. If the generated number is less than or equal to $\Pr[S_t = 1 | S_{t+1}, \widetilde{r}_T]$, we set $S_t = 1$. Otherwise, we set $S_t = 2$.

6.2 Generating transition probabilities conditional on \widetilde{S}_T

We will use beta distributions as conjugate priors for the transition probabilities. Then it can be shown that the posterior distributions of p_{ii} are given by $p_{ii} | \widetilde{S}_T \sim \text{beta}(u_{ii} + n_{ii}, \bar{u}_{ii} + \bar{n}_{ii})$, $i = 1, 2$, where u_{ii} and \bar{u}_{ii} are known from the prior distributions ($p_{ii} \sim \text{beta}(u_{ii}, \bar{u}_{ii})$ a priori). n_{ij} , $i, j = 1, 2$ is defined as the total number of transitions from state $S_{t-1} = i$ to $S_t = j$, $t = 2, 3, \dots, T$. \bar{n}_{ij} is defined as the number of transitions from state $S_{t-1} = i$ to $S_t \neq j$. Once the p_{ii} are generated, generating p_{ij} is straightforward: $p_{ij} = \bar{p}_{ij} * (1 - p_{ii})$, where $\bar{p}_{ij} = \Pr[S_t = j | S_{t-1} = i, S_t \neq i]$, for $i \neq j$.

6.3 Generating $\widetilde{\sigma}^2$, conditional on $\widetilde{\mu}, \widetilde{\phi}, \widetilde{r}_T, \widetilde{S}_T$

To generate σ_j^2 , $j = 1, 2$, we can redefine $\sigma_{s_t}^2$ as follows: $\sigma_{s_t}^2 = \sigma_1^2 (1 + S_{2t} h_2)$. Then we can generate σ_1^2 conditional on h_2 , by first dividing e_t by $\sqrt{(1 + S_{2t} h_2)} = U$, and then choosing an inverse gamma distribution as the prior for σ_1^2 ($IG(\frac{v_1}{2}, \frac{\delta_1}{2})$).

Then the posterior for σ_1^2 is given by $\sigma_1^2 | \widetilde{r}_T, \widetilde{S}_T, h_2 \sim IG(\frac{v_1 + T}{2}, \frac{\delta_1 + \sum_{t=1}^T (e_t^*)^2}{2})$,

where v_1 and δ_1 are known hyperparameters of the prior distribution, and e_t^* is e_t divided by U defined above. Then we generate $\bar{h}_2 = 1 + h_2$, conditional on σ_1^2 to find σ_2^2 .

6.4 Generating $\tilde{\mu}$, conditional on $\tilde{\sigma}^2, \tilde{\phi}, \tilde{r}_T, \tilde{S}_T$

It is straightforward to derive the posterior distribution of $\mu = [\mu_1 \mu_2]'$, given an appropriate prior distribution such as the normal: $\mu|\tilde{\sigma}^2, \tilde{\phi} \sim N(a_0, A_0)$, where a_0 and A_0 are known hyperparameters of the prior distribution. Then the posterior is: $\mu|\tilde{\sigma}^2, \tilde{\phi}, \tilde{r}_T, \tilde{S}_T \sim N(a_1, A_1)$ where $a_1 = (A_0^{-1} + S_T^* S_T^*)^{-1}(A_0^{-1} a_0 + S_T^* r_T^*)$, $A_1 = (A_0^{-1} + S_T^* S_T^*)^{-1}$, where $S_{it}^* = S_{it}$, and S_T^* is the matrix form of S_{it}^* , each element divided by σ_{S_t} to correct for heteroskedasticity, r_T^* is the matrix form of r_t^* , each element divided by σ_{S_t} to correct for heteroskedasticity, where $r_t^* = r_t$.

6.5 Generating $\tilde{\phi}$, conditional on $\tilde{\sigma}^2, \tilde{\mu}, \tilde{r}_T, \tilde{S}_T$

If we let $r_t = \mu_{S_t} + X\phi_{S_t} + v$, and then divide it by σ_{S_t} , then given the a priori distribution for $\tilde{\phi}$ as $\tilde{\phi}|\tilde{\sigma}^2, \tilde{\mu} \sim N(b_0, B_0)$, where b_0 and B_0 are known hyperparameters of the prior distribution, the posterior can be written as $\tilde{\phi}|\tilde{\sigma}^2, \tilde{\mu}, \tilde{r}_T, \tilde{S}_T \sim N(b_1, B_1)$, where $b_1 = (B_0^{-1} + X'X)^{-1}(B_0^{-1} b_0 + X'r)$, $B_1 = (B_0^{-1} + X'X)^{-1}$.

6.6 Prior Elicitation

We make use of the following prior hyperparameters.

For the hyperparameters of the prior for the transition probabilities (where the prior is a beta distribution) we use:

$$p_{ii} \sim \text{beta}(u_{ii}, \bar{u}_{ii}) \Rightarrow u_{ii} = 0.1, \bar{u}_{ii} = 0.1.$$

For the hyperparameters of the prior for the variances (where the prior is an inverted gamma distribution) we use:

$$\sigma_i^2 \sim IG(\frac{v_i}{2}, \frac{\delta_i}{2}) \Rightarrow v_i = 0, \delta_i = 0.$$

For the hyperparameters of the prior for the means (where the prior is a normal distribution) we use:

$$\mu \sim N(a_0, A_0) \Rightarrow a_0 = 0.04, A_0 = 0|0.1.$$

For the hyperparameters of the prior for the coefficients (where the prior is a normal distribution) we use:

$$\phi \sim N(b_0, B_0) \Rightarrow b_0 = 0, B_0 = 0|0.$$

7 References

Ang, A., and G. Bekaert, 2002, "International Asset Allocation with Regime Shifts," *Review of Financial Studies*, 15, 4, 1137-1187.

Barberis, Nicholas, 2000, "Investing for the Long Run when Returns are Predictable," *Journal of Finance*, Vol. LV, No.1.

Bawa, V., S. Brown, and R. Klein, 1979, "Estimation Risk and Optimal Portfolio Choice" (North Holland, Amsterdam).

Beller, K., J. Kling, and M. Levinson, 1998, "Are Industry Stock Returns Predictable?," *Financial Analysts Journal*, Vol. 54, No. 5, Sep/Oct, 42-57.

Bodie, Zvi, 1995, "On the Risk of Stocks in the Long Run," *Financial Analysts Journal* 51, 18-22.

Capaul, Carlo, 1999, "Asset Pricing Anomalies in Global Industry Indexes," *Financial Analysts Journal*, Vol. 55, No. 4, Jul/Aug, 17-37.

Cavaglia, S., and V. Moroz, 2002, "Cross-Industry, Cross-Country Allocation," *Financial Analysts Journal*, Vol. 58, Issue 6, Nov/Dec, p.78.

Fama, E.F., and K.R. French, 1988a, "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, Vol. 96, No. 2, April, 246-73.

Ferson, W.E., and C.R. Harvey, 1991, "The Variation of Economic Risk Premiums," *Journal of Political Economy*, Vol. 99, No. 2, April, 385-415.

Frost, P., and J. Savarino, 1986, "An empirical Bayes Approach to Efficient Portfolio Selection," *Journal of Financial and Qualitative Analysis* 21, 293-305.

Hamilton, James, "A New Approach to the Economic Analysis of Non-Stationary Time Series and The Business Cycle," *Econometrica*, March 1989, 57, 357-84.

Hamilton, J.D., and R. Susmel, 1994, "Autoregressive Conditional Heteroskedasticity and Changes in Regime," *Journal of Econometrics* 64, 307-333.

Jobson, J.D., and R. Korkie, 1980, "Estimation for Markowitz Efficient Portfolios," *Journal of the American Statistical Association* 75, 544-554.

Jorion, Phillippe, 1985, "International Portfolio Diversification with Estimation Risk," *Journal of Business* 58, 259-278.

Kandel, S., and R. Stambaugh, 1996, "On the Predictability of Stock Returns: An Asset Allocation Perspective," *Journal of Finance* 51, 385-424.

Kim, C.J., and C. R. Nelson, "State-Space Models with Regime Switching: Classical and Gibbs-sampling Approaches with Applications," 1998, MIT Press.

Lo, A., and A.C. MacKinlay, 1996, "Maximizing Predictability in the Stock and Bond Markets," NBER Working Paper, #5027, February 1995.

Massimo G., and A. Timmermann, 2005, "Strategic Asset Allocation and Consumption Decisions Under Multivariate Regime Switching," Working Papers 2005-002, Federal Reserve Bank of St. Louis.

Merton, Robert, 1969, "Lifetime Portfolio Selection: The Continuous-Time Case," *Review of Economics and Statistics* 51, 247-257.

Moskowitz, T.J., and M. Grinblatt, 1999, "Do Industries Explain Momentum?," *Journal of Finance*, Vol. 54, No. 4, August, 1249-90.

Perez-Quiros, G., and A. Timmermann, 2001, "Business Cycle Asymmetries in Stock Returns : Evidence from Higher Order Moments and Conditional Densities," Working Paper Series 58, European Central Bank.

Pettenuzzo, D., and A. Timmermann, "Optimal Asset Allocation under Structural Breaks," job market paper.

Samuelson, Paul, 1969, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *Review of Economics and Statistics* 51, 239-246.

Samuelson, Paul, 1994, "The Long-Term Case for Equities," *Journal of Portfolio Management*, Fall, 15-24.

Schwert, G. William, 1989, "Business Cycles, Financial Crises, and Stock Volatility," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 31, 83-126.

Siegel, Jeremy, 1994, "Stocks for the Long Run" (Richard D. Irwin, Burr Ridge, Ill.).

Sorenson, E., and T. Burke, 1986, "Portfolio Returns from Active Industry Group Rotation," *Financial Analysts Journal*, Vol. 46, No. 5, Sept/Oct, 43-50.

Turner, C. M., R. Startz, and C. R. Nelson, 1989, "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market," *Journal of Financial Economics*, Vol. 25, 3-22.

Schaller, H. and S. Van Norden, 1997, "Regime Switching in Stock Market Returns," *Applied Financial Economics*, Taylor and Francis Journals, vol. 7(2), pages 177-91.

Descriptive Statistics 1

	1926/12-2003/12			1953/01-2003/12		
	T-bill	High Tech	Manuf	T-bill	High Tech	Manuf
Mean	3.684	4.875	6.201	5.063	4.760	7.263
Standard Error	0.102	2.366	2.218	0.115	3.028	2.362
Median	3.303	4.446	7.539	4.701	5.060	7.113
Standard Deviation	3.102	20.772	19.474	2.851	21.624	16.868
Kurtosis	1.448	13.673	3.498	2.014	16.492	1.542
Skewness	1.064	-1.237	-0.223	1.139	-1.642	0.088
Minimum	0.000	-724.784	-356.605	0.356	-724.784	-198.189
Maximum	18.053	293.098	328.454	18.053	293.098	274.851
Count	925	925	925	612	612	612

Figure 1: This table gives the descriptive statistics for 1 month T_bill rates, value weighted high tech and manufacturing sector returns. All returns are continuously compounded and in annual terms. The data covers months between 1926/12-2003/12 and months between 1953/01-2003/12 in two different samples. We give the descriptive statistics for both datasets.

Descriptive Statistics 2

	1973/01-2003/12			
	T-bill	High Tech	Manuf	Nasdaq
Mean	6.197	2.896	7.157	10.225
Standard Error	0.152	4.532	3.210	4.188
Median	5.684	5.974	4.686	18.144
Standard Deviation	2.925	25.232	17.875	23.316
Kurtosis	1.565	13.748	1.680	2.695
Skewness	0.940	-1.772	0.077	-0.865
Minimum	0.809	-724.784	-198.189	-379.428
Maximum	18.053	293.098	274.851	238.407
Count	372	372	372	372

Figure 2: This table gives the descriptive statistics for 1 month T_bill rates, Nasdaq value weighted portfolio returns, value weighted high tech and manufacturing sector returns. All returns are continuously compounded and in annual terms. The data covers months between 1973/01-2003/12.

Descriptive Statistics 3

	1926/12-2003/12	
	<i>Dividend Yield High Tech</i>	<i>Dividend Yield Manufacturing</i>
Mean	0.040	Mean 0.042
Standard Error	0.001	Standard Error 0.000
Median	0.041	Median 0.040
Standard Deviation	0.019	Standard Deviation 0.013
Kurtosis	0.147	Kurtosis 3.266
Skewness	0.274	Skewness 1.129
Minimum	0.004	Minimum 0.018
Maximum	0.118	Maximum 0.124
Count	924	Count 924

Figure 3: This table gives the descriptive statistics for dividend yields for high tech and manufacturing sectors. The data covers months between 1926/12-2003/11.

Posterior Means for a Linear Model of Excess Stock Returns			
		μ	σ^2
(1926/12-2003/12)			
	High Tech	0.0043	0.0068
StdDev		0.0087	0.0003
	Manufacturing	0.0089	0.0060
StdDev		0.0081	0.0003
(1973/01-2003/12)			
	Nasdaq	0.0145	0.0086
StdDev		0.0153	0.0006
(1953/01-2003/12)			
	High Tech	-0.0010	0.0074
StdDev		0.0111	0.0004
	Manufacturing	0.0078	0.0045
StdDev		0.0086	0.0003

Figure 4: The parameter estimates in this table are based on the model: $r_t = \mu + \varepsilon_t$, where r_t is the continuously compounded excess return in month t and $\varepsilon_t \sim N(0, \sigma^2)$. The table gives the mean and standard deviation (below the mean) of each parameter's posterior distribution for the three different time periods we analyze in this paper. All the estimates are scaled up by ten to be comparable to the following figures.

Posterior Means for a Two State Markov Switching Model of Excess Stock Returns									
	Nasdaq (1973/01-2003/12)			Manuf (1926/12-2003/12)			High Tech (1926/12-2003/12)		
	Mean	StdDev	Median	Mean	StdDev	Median	Mean	StdDev	Median
sig_1	0.023	0.005	0.023	0.022	0.003	0.021	0.020	0.004	0.019
sig_2	0.004	0.000	0.004	0.003	0.000	0.003	0.002	0.000	0.002
mu_1	-0.106	0.057	-0.104	-0.057	0.038	-0.053	-0.024	0.022	-0.019
mu_2	0.051	0.014	0.051	0.019	0.007	0.019	0.008	0.007	0.008
Pr(St=1)	0.025	0.155	0.000	0.179	0.383	0.000	0.137	0.344	0.000
Pr(St=2)	0.976	0.155	1.000	0.821	0.383	1.000	0.863	0.344	1.000
Transition Probabilities									
		Regime 1	Regime 2		Regime 1	Regime 2		Regime 1	Regime 2
StdDev	Regime 1	0.884	0.116	Regime 1	0.893	0.107	Regime 1	0.824	0.176
	Regime 2	0.049		Regime 2	0.045		Regime 2	0.056	
StdDev	Regime 1	0.035	0.965	Regime 1	0.018	0.982	Regime 1	0.069	0.931
	Regime 2	0.017		Regime 2	0.008		Regime 2	0.033	

Figure 5: The parameter estimates in this table are based on a two state Markov Switching model of continuously compounded monthly excess returns. The table gives the mean, standard deviation and median of each parameter's posterior distribution. The results are given for two different time periods: 1973/01-2003/12 for the Nasdaq portfolio and 1926/12-2003/12 for the manufacturing and high tech sectors. The means and standard deviations are scaled up by ten.

Posterior Means for a Two State Markov Switching Model of Excess Stock Returns						
	Manuf (1953/01-2003/12)			High Tech (1953/01-2003/12)		
	Mean	StdDev	Median	Mean	StdDev	Median
sig_1	0.016	0.005	0.016	0.036	0.010	0.034
sig_2	0.003	0.000	0.003	0.004	0.000	0.004
mu_1	-0.056	0.049	-0.045	-0.076	0.059	-0.063
mu_2	0.013	0.008	0.013	0.004	0.009	0.004
Pr(St=1)	0.292	0.455	0.000	0.027	0.162	0.000
Pr(St=2)	0.708	0.455	1.000	0.973	0.162	1.000
Transition Probabilities						
		Regime 1	Regime 2		Regime 1	Regime 2
	Regime 1	0.760	0.240	Regime 1	0.839	0.161
StdDev		0.126			0.081	
	Regime 2	0.030	0.970	Regime 2	0.022	0.978
StdDev		0.019			0.011	

Figure 6: The parameter estimates in this table are based on a two state Markov Switching model of continuously compounded monthly excess returns. The table gives the mean, standard deviation and median of each parameter's posterior distribution. The results are given for the time period: 1953/01-2003/12 for the manufacturing and high tech sectors. The means and standard deviations are scaled up by ten.

Posterior Means With Two Predictors and a Two State MSM of Excess Returns						
	Manuf (1926/12-2003/12)			High Tech (1926/12-2003/12)		
	Mean	StdDev	Median	Mean	StdDev	Median
phi_1	-0.390	0.300	-0.396	-0.267	0.233	-0.270
phi_2	0.239	0.061	0.239	0.062	0.038	0.062
sig_1	0.022	0.003	0.022	0.020	0.003	0.020
sig_2	0.003	0.000	0.003	0.002	0.000	0.002
mu_1	-0.165	0.057	-0.163	-0.036	0.026	-0.032
mu_2	-0.066	0.027	-0.066	-0.010	0.019	-0.010
Pr(St=1)	0.276	0.447	0.000	0.152	0.359	0.000
Pr(St=2)	0.725	0.447	1.000	0.848	0.359	1.000
Transition Probabilities						
		Regime 1	Regime 2		Regime 1	Regime 2
	Regime 1	0.886	0.114	Regime 1	0.847	0.153
StdDev		0.044			0.056	
	Regime 2	0.020	0.980	Regime 2	0.055	0.945
StdDev		0.008			0.027	

Figure 7: The parameter estimates in this table are based on a model with two predictor variables (T-bill rate and dividend yield) and a two state Markov Switching model of continuously compounded monthly excess returns. The table gives the mean, standard deviation and median of each parameter's posterior distribution. Phi_1 is the coefficient for the T-bill rate and phi_2 is the coefficient for the dividend yield. The results are given for the time period: 1926/12-2003/12 for the manufacturing and high tech sectors. The means and standard deviations are scaled up by ten.

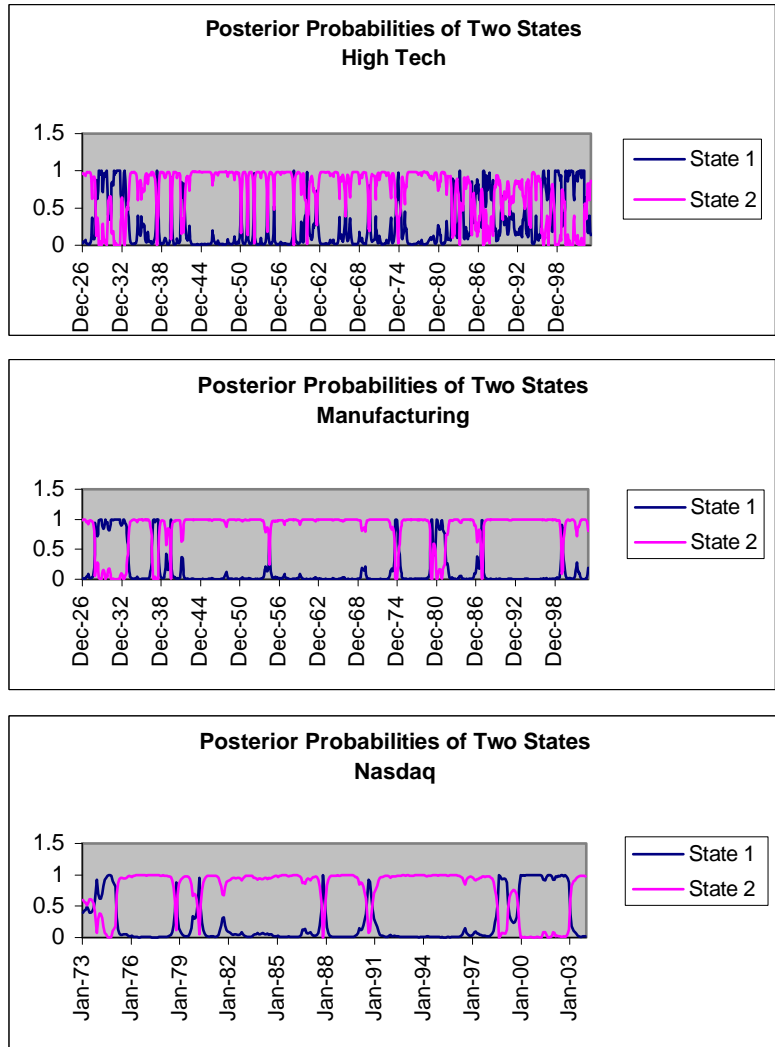


Figure 8: This figure gives the posterior probabilities of two states for months between 1926/12-2003/12 for the high tech and manufacturing sectors and for months between 1973/01-2003/12 for the Nasdaq portfolio.

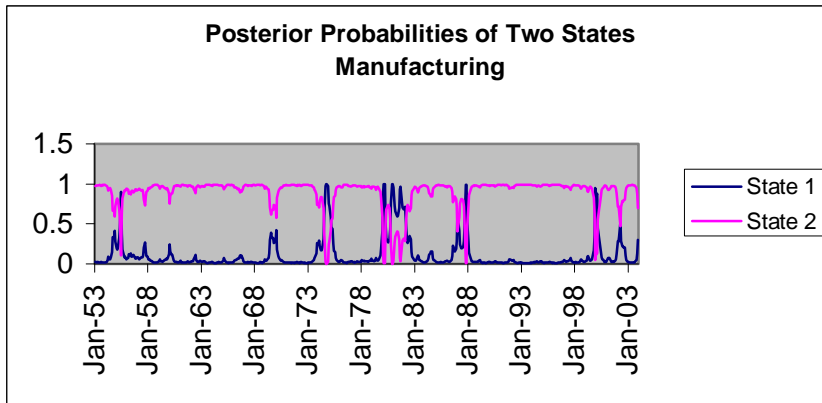
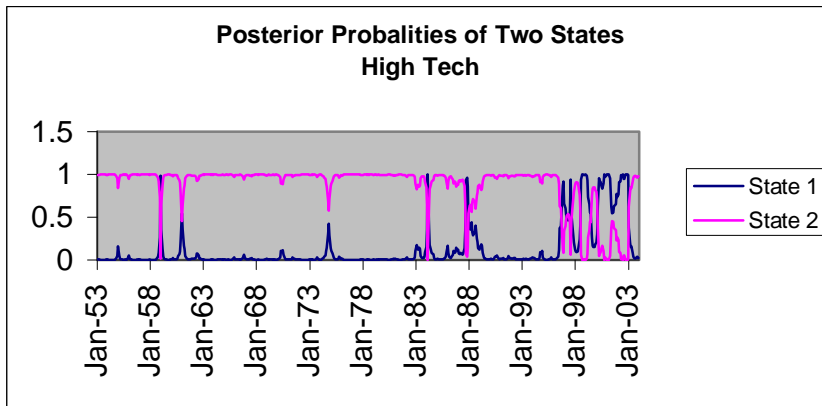


Figure 9: This figure gives the posterior probabilities of two states for months between 1953/01-2003/12 for the high tech and manufacturing sectors.

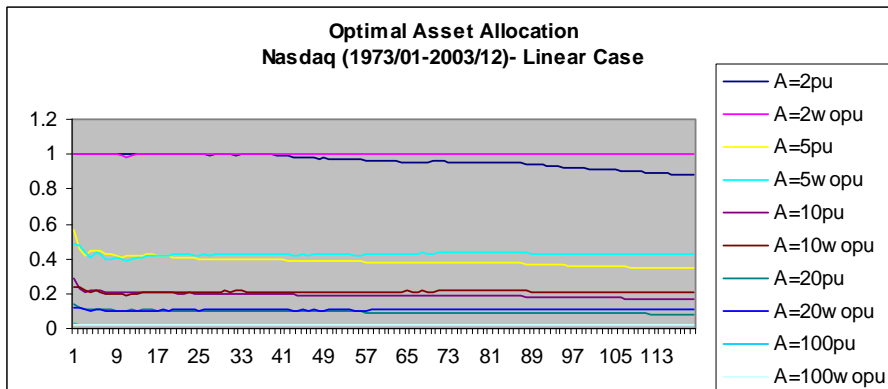
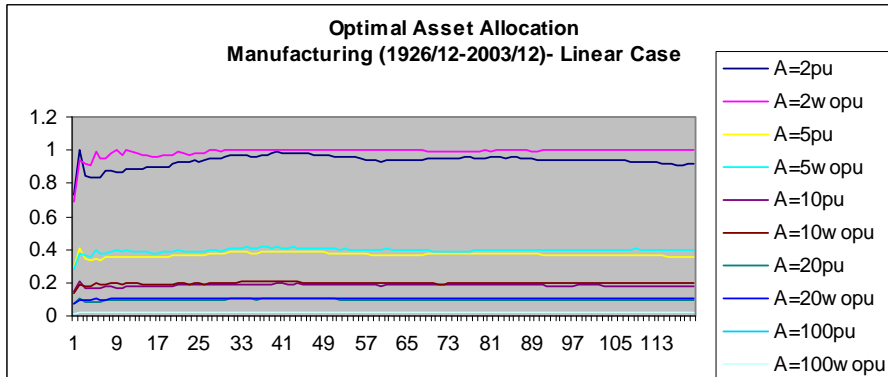
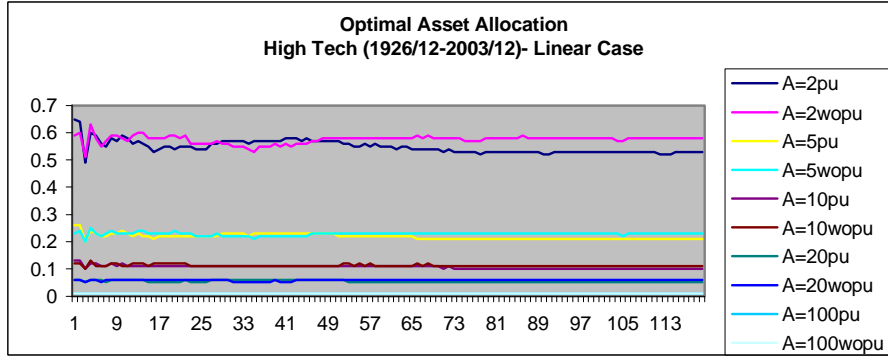


Figure 10: This figure gives the optimal allocation to high tech, manufacturing, and nasdaq stocks when the returns for each series follow a linear model. The results are given for different risk aversion levels and for a 10 year investment horizon. The figures show both the case with and the case without parameter uncertainty. A=2pu for instance stands for a risk aversion level of 2 and a case with parameter uncertainty. A=2wopu stands for a risk aversion level of 2 and a case without parameter uncertainty.

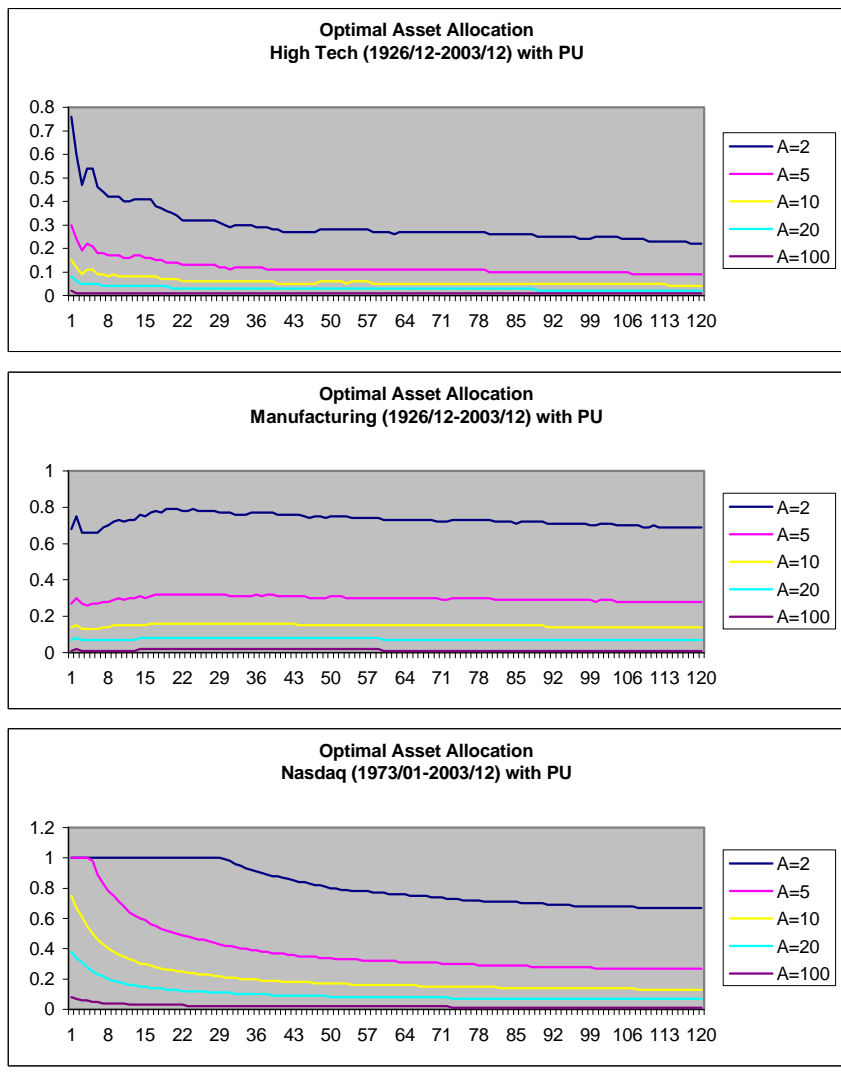


Figure 11: This figure shows the optimal asset allocation (with parameter uncertainty) of each of the three series, Nasdaq, high tech and manufacturing sector portfolios versus the risk free asset. The model for returns for each series is the two state Markov Switching model. The sample period is 1926/12-2003/12 for the two sectors and 1973/01-2003/12 for Nasdaq.

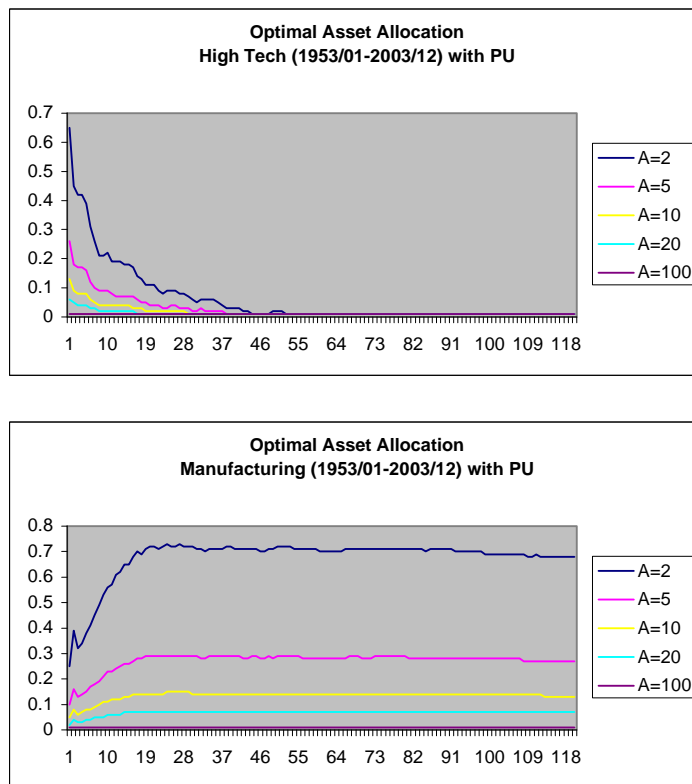


Figure 12: This figure shows the optimal asset allocation (with parameter uncertainty) of each of the two sector (high tech and manufacturing) portfolios versus the risk free asset. The model for returns for each series is the two state Markov Switching model. The sample period is 1953/01-2003/12.

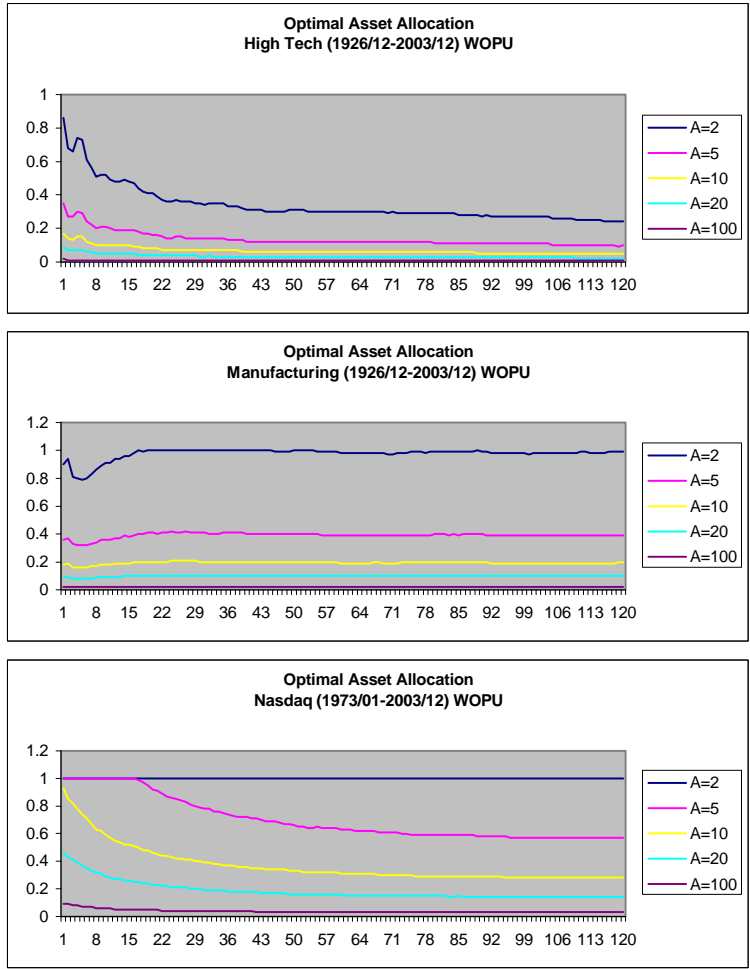


Figure 13: This figure shows the optimal asset allocation (without parameter uncertainty) of each of the three series, Nasdaq, high tech and manufacturing sector portfolios versus the risk free asset. The model for returns for each series is the two state Markov Switching model. The sample period is 1926/12-2003/12 for the two sectors and 1973/01-2003/12 for Nasdaq.

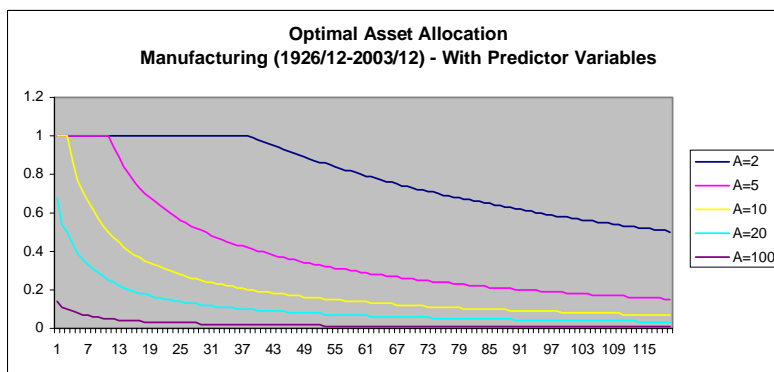
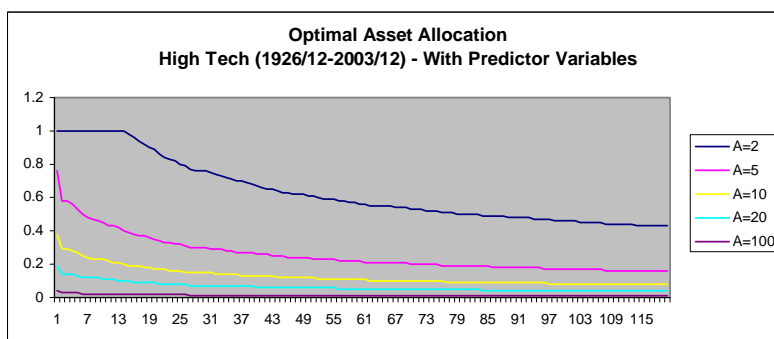
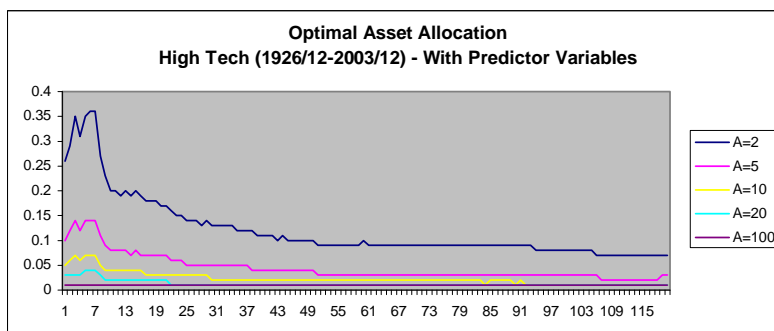


Figure 14: This figure gives the optimal asset allocation (with parameter uncertainty) of high tech and manufacturing stocks versus the risk free asset with a two state Markov Switching model and dividend yield and T-bill rate as predictor variables. The sample mean of the dividend yield and T-bill rate is utilized in the graph at the top, the sample max of the dividend yield and T-bill rate is utilized in the graph in the middle, and again the sample max of the dividend yield and T-bill rate is utilized in the graph at the bottom. The top two graphs are utilizing the high tech sector data and the bottom graph is utilizing the manufacturing sector data.

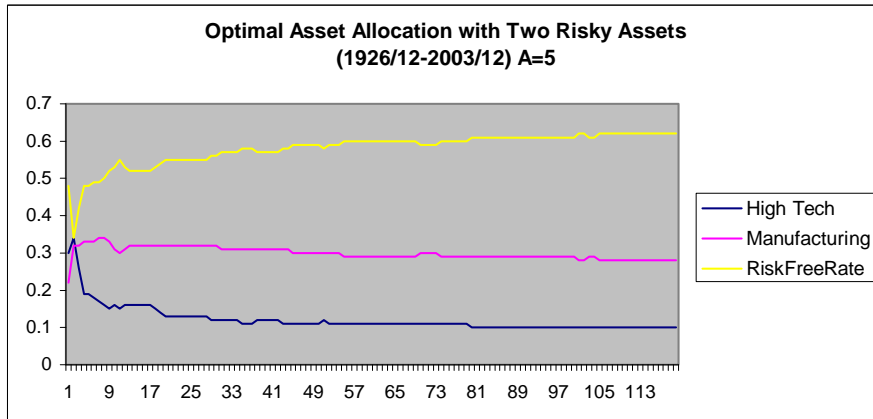
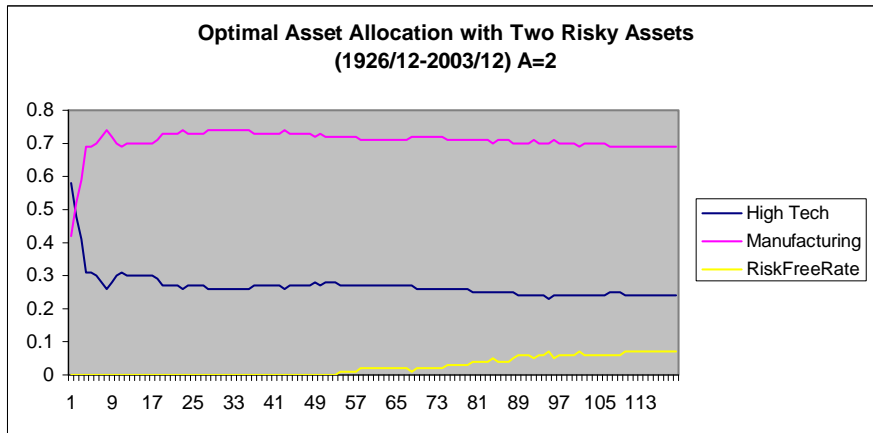


Figure 15: This figure gives the optimal asset allocation with two risky assets (high tech and manufacturing sector portfolios) versus the risk free asset. The returns for each series follow a two state Markov Switching model, with parameter uncertainty taken into account. The sample period utilized is 1926/12-2003/12 and the graphs show the allocations with different risk aversion levels: $A=2$, $A=5$.

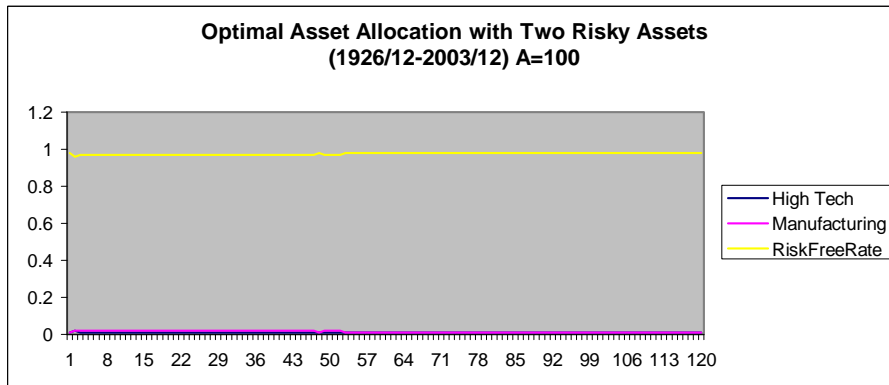
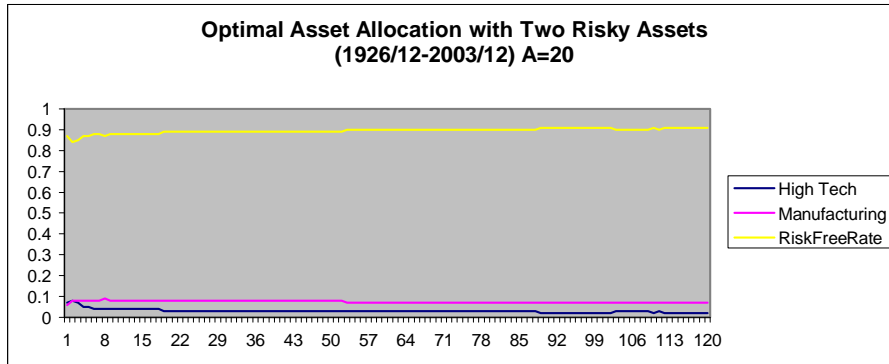
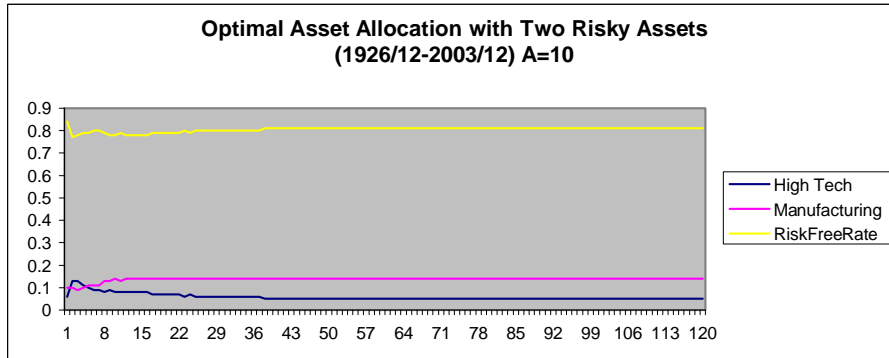


Figure 16: This figure gives the optimal asset allocation with two risky assets (high tech and manufacturing sector portfolios) versus the risk free asset. The returns for each series follow a two state Markov Switching model, with parameter uncertainty taken into account. The sample period utilized is 1926/12-2003/12 and the graphs show the allocations with different risk aversion levels: $A=10$, $A=20$, and $A=100$.