Final Exam, Econ 214C

Winter 1998

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Please attempt to answer three questions. Each question carries a third of the total grade.

1. Market Completeness, Arbitrage Pricing.

(i) Define a dynamically complete market in the context of a finite-state model. Explain how a dynamic trading strategy can complete a market that does not have a full set of time- and state-contingent Arrow-Debreu securities.

(ii) Prove that under no arbitrage there exists a pricing kernel Q_{t+1} such that for all assets

$$E[Q_{t+1}R_{i,t+1}] = 1,$$

where $R_{i,t+1}$ is the gross rate of return on the $i'th$ asset.

(iii) The no-arbitrage condition implies that some function of asset prices and dividends will follow a martingale process under the risk-neutral probability measure. Demonstrate this result.

(iv) Explain the relationship between the risk-neutral density and the (empirical) density of observed asset payoffs. Which states of the world does the risk-neutral density over-weight relative to the empirical density?

(v) Briefly characterize the main properties of a rational expectations equilibrium in the following cases:

(a) an Arrow-Debreu economy with complete markets
(b) a dynamically complete market (Radner equilibrium)
(c) an (dynamically and statically) incomplete market.

2. Equity Premium Puzzle. Hansen-Jagannathan Bounds.

(i) State and explain what is meant by the equity premium and the risk-free rate puzzle.

(ii) Discuss two of the following directions that the finance literature has taken to resolve the equity premium puzzle:

(a) market frictions/heterogenous agents
(b) non-standard preferences
(c) estimation uncertainty

(iii) State the Hansen-Jagannathan bound and explain in words what it measures.

(iv) Suppose a representative agent has utility function $U(C_{t+1}) = \beta C_{t+1}^{\alpha}$and maximizes expected utility. Explain how you could use GMM estimation to obtain estimates of α and the rate of time preference parameter (β). In particular set up the orthogonality condition implied by the agent's first order condition and comment on how over-identifying restrictions could be tested.

3. Present Value Model and Excess Volatility.

(i) State and derive the variance bound used in the excess volatility literature. What exactly is meant by the statement that stock prices are 'excessively volatile'? Is this a testable proposition?

(ii) Suppose that, each period, the real dividend takes a value D_h with probability 0.5 and a value D_l with probability 0.5. Verify for this case that the variance bound holds.

(iii) Explain the problems arising from using Shiller's original variance bound due to

(a) non-stationary prices
(b) serially correlated dividends
(c) time-varying risk premia
(d) learning effects

(iv) Explain the relationship between variance bound tests and regression tests of predictability of stock returns.

(v) State the condition defining a speculative bubble in an asset price. Does the rational expectations assumption rule out the presence of a speculative bubble in asset prices?

4. Mean Reversion in Asset Prices.

Suppose that the logarithm of the stock price cum dividends (p_t) follows the process

$$
\begin{aligned}
p_t &= y_t + z_t \\
z_t &= \phi z_{t-1} + \varepsilon_t, |\phi| < 1, \\
y_t &= \mu + y_{t-1} + u_t,
\end{aligned}
$$

where ε_t and u_t are uncorrelated, white noise processes.

(i) derive an expression for the least squares estimator of the coefficent β_T in the regression

$$r_{t:t+T} = \alpha_T + \beta_T r_{t-T:t} + e_{t,T},$$

where T is the return horizon and $r_{t:t+T} = p_{t+T} - p_t$, $r_{t-T:t} = p_t - p_{t-T}$.

(ii) Plot β_T as a function of T and explain which patterns in β_T will be obtained as a function of the size of the z_t and y_t components in the stock price.

(iii) Define the variance ratio statistic and derive an expression for this ratio in the above model. Plot it as a function of T.

(iv) Explain the relationship between the variance ratio and regression tests of mean reversion.

(v) Comment on some statistical problems associated with testing for 'mean reversion' in stock market prices and briefly describe the empirical evidence that has emerged on this issue.

5. Nonlinearities in Asset Prices. Random Walk Model.

Campbell, Lo and MacKinlay (1997) characterize different versions of the random walk hypothesis and the martingale model based on the condition

$$Cov(f(r_t), g(r_{t+k})) = 0, \text{ for all } k \geq 1.$$

(i) For each of the models (a) - (c), give an example of a data generating return process that violates the particular model without violating the model succeeding it

(a) returns are independently and identically distributed (RW1)

(b) returns are independently but not identically distributed (RW2)

(c) returns follow a martingale

(d) returns are serially uncorrelated (RW3)

(ii) Choose any two of the models (a) - (d) and explain which type of (estimation) strategy could be used to test the model.

(iii) State the equation for a GARCH(1,1) model and explain the condition under which the model is covariance stationary. What determines the persistence of the conditional volatility?

(iv) Derive the log-likelihood function of the GARCH(1,1) model and explain how you would estimate its parameters. Comment on the properties of the resulting estimates.

(v) Show that the coefficient of kurtosis generated by a Gaussian mixture model is higher than that of a single Gaussian distribution.