Final Exam, Econ214A

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Allan Timmermann

Please attempt to answer 3 of the 4 questions. Each question counts for a third of the total grade.

1. Binomial Lattice, Option Pricing
A firm's stock price follows the binomial lattice

		S_{uu}
	S_u	
S		S_{du}
	S_d	
		S_{dd}
time 0	time 1	time 2

In addition to the stock, a risk-free bond with a return of r is also traded.

(i) At time 0 show how you can replicate the period-1 payoff on a European call option expiring at time 2 by means of a portfolio invested in the risk-free bond and the underlying stock.
(ii) What is the time 0 value of this call option?
(iii) How would your answer change if, instead, the call option was American?
(iv) Show how the binomial lattice model can be generalized to the case with n time periods. How many different paths of the stock price need to be considered in pricing a European call?
(v) Explain how the Black-Scholes European option pricing formula emerges as the limiting case of the binomial lattice when the number of steps, n, goes to infinity. How do the parameters of the binomial lattice need to be modified, and what is the distribution of the returns on the underlying stock in this limit?

2. Diffusion Processes
You are considering pricing a derivative contract, $F(S_t, D_t, t)$, which depends on the price of some underlying asset (S_t) as well as on an additional random factor, D_t :

$$\begin{aligned} dS_t &= a_t d_t + \sigma_t dW_t, \\ dD_t &= a_t^* d_t + \sigma_t^* dW_t^* \end{aligned}$$

where a_t, σ_t, a_t^* and σ_t^* are bounded functions of variables known at time t, and dW_t, dW_t^* are increments to standard Wiener processes.

(i) Suppose that dW_t and dW_t^* are uncorrelated. Demonstrate that a portfolio composed of the underlying asset and the derivative, $F(.)$, cannot be used to completely eliminate risk.

(ii) Now suppose that dW_t and dW_t^* are perfectly correlated and that, in fact, $dW_t = dW_t^*$. Set up a risk-free hedge portfolio composed of the stock and the derivative and derive the SDE which the derivative price $F(.)$ must satisfy under no arbitrage.

(iii) Explain what is meant by 'dynamic hedging'. How can you hedge a call option through a dynamic hedging scheme?

(iv) What are the main differences between a Wiener process and a Poisson process?

(v) Suppose you expect the underlying asset price to be very volatile in the near future. Describe an investment strategy in options which would benefit if these beliefs turn out to be correct.

3. Arbitrage Pricing. Investor Preferences.

You are in a discrete single-period economy with N securities and K states of nature. The $(K \bullet N)$ asset payoff matrix is D and the $(N \bullet 1)$ price vector is $\mathbf{p}$.

(i) Suppose an additional asset, $N+1$, with price p_{N+1} and payoff vector $\mathbf{D}_{N+1}$ is introduced into this economy. Are there any restrictions on the price of this asset resulting from no arbitrage? Distinguish between the complete and incomplete market cases.

(ii) In this economy, what are the conditions under which an equilibrium price measure (risk-neutral probability measure) exists?

(iii) What are the conditions under which the equilibrium price measure is unique?

(iv) Explain why, in an incomplete market economy, competitive equilibria aren't generally Pareto-optimal.

(v) Establish a condition, similar to second order stochastic dominance, for a non-satiated, *risk-loving* investor to prefer one risky asset $\widetilde{X}$ to another risky asset $\widetilde{Y}$ (such that $E[\widetilde{X}] = E[\widetilde{Y}]$). You may assume that the return distributions are bounded on $[0; 1]$.

4. CAMP-APT Models

(i) State the Sharpe-Lintner CAPM with a risk-free asset and explain how you would estimate and test this model when returns are joint multivariate normally distributed.

(ii) Next state the Black CAPM without a risk-free asset. Again explain how you would estimate and test the model under joint normality.

(iii) Compare the testable implications of the CAPM to those of the APT model.

(iv) What is meant by the 'rotational indeterminacy' problem in tests of the APT model?

(v) Suppose that an econometric analysis leads to the conclusion that there are two factors explaining the cross-section of expected returns. Does this mean we can reject the CAPM?