Final Exam, ECON214A

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Please attempt to answer all questions. Each question carries 25% of the total grade.

1. Market Completeness. Risk Neutral Pricing.

Consider a two-period economy with K states and N assets represented by a $K \bullet N$ payoff matrix D and an $N \bullet 1$ price vector $\mathbf{p}$.

(i) State the condition under which the market is complete.

(ii) Define an 'arbitrage trading strategy'. State the condition on asset prices and payoffs such that arbitrage opportunities are ruled out.

(iii) Prove that arbitrage trading strategies do not exist in equilibrium.

(iv) Explain why the equilibrium allocation in an incomplete financial market need not be pareto optimal.

Now suppose you are in the special case with 3 states and 3 assets, the payoff matrix of which is given by (where D_{ij} is the payoff in state i on asset j)

$$D = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & 7 \\ 1 & 0 & 0 \end{bmatrix}$$

The associated price vector is $\mathbf{p} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$.

(v) Does there exist an equilibrium price measure $(Q(\omega_j))$ for this economy? If so, derive this measure.

(vi) Is the equilibrium price measure derived in (v) unique?

(vii) In this economy, what is the price of an asset with a payoff vector equal to $\begin{pmatrix} 5 & 10 & 15 \end{pmatrix}'$?

2. CAPM-APT.

(i) Explain the CAPM argument leading to the conclusion that the market portfolio is on the efficient frontier. Be careful to state the assumptions made on agents' probability beliefs and preferences.

(ii) Explain the errors-in-variables problem in empirical tests of the CAPM. Describe how grouping individual assets into portfolios may lead to a consistent estimator of the risk premium and comment on the drawback of this approach.

(iii) Describe the Jensen and Sharpe performance measures. Under which set of assumptions are these suitable measures of investment performance?

(iv) State the APT model and carefully explain the underlying assumptions on the covariance structure for the factors and the residual returns.

(v) Show that the no-arbitrage assumption means that the vector of mean asset returns is a linear function of the factor exposures (the betas).

3. Option Pricing. Efficient Market Hypothesis.

(i) Show that a portfolio of long positions in (simultaneously expiring) options is worth at least as much as an option on the portfolio of the underlying stocks.

(ii) State and prove put-call parity for European options.

(iii) Demonstrate how options can be used to complete financial markets. How would you price such options?

(iv) Explain the difference between 'pricing assets by utility' and 'pricing assets by no-arbitrage' and explain which types of assets the two approaches apply to.

(v) Suppose you have come up with a trading strategy that generates higher mean returns than returns from a buy-and-hold strategy in the market portfolio. Carefully explain what are the implications of this finding for the efficient market hypothesis.

4. Diffusion Processes.

(i) Give the continuous-time equations for

(a) arithmetic Brownian motion

(b) geometric Brownian motion

(c) Ornstein-Uhlenbeck process.

Describe the main properties of each process and explain which type of asset whose price you could characterize by means of the process.

(ii) Describe the difference between a jump and a diffusion process. Why can diffusion risk, but typically not jump risk, be hedged?

(iii) Let S follow a geometric Brownian motion process and suppose $f = \ln(S)$. Use Ito's lemma to derive the process followed by f. In particular derive the drift and volatility of f.

(iv) Explain what is meant by 'non-linear mean reversion' for the short-term spot rate in the RFS paper by Ait-Sahalia. Also describe the diffusion process he proposes to capture this effect.