Assignment 4, ECON 214A

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1. The expected return on a portfolio in a sequence of economies with n assets is given by

$$\rho^n + \sum_i \sum_k \omega_i(n) b_{ik} \gamma_k^n + \sum_i \omega_i(n) c_i^n,$$

where $(\omega_i(n))_{i=1}^n$ are the portfolio weights, γ_k^n are risk premia and c_i^n are pricing errors. Under the APT the pricing errors satisfy $\sum_i (c_i^n)^2 < A < \infty$. Consider a well-diversified portfolio satisfying

$$\lim_{n \to \infty} n \bullet \sum_{i=1}^n \omega_i^2(n) \leq C < \infty.$$

Prove that, under no arbitrage, the expected return on a well-diversified portfolio is given exactly with zero error by the APT pricing model.

2.

(i) Using the data from Assignment 3, compute the first two principal components of the excess return data. Standardize the eigenvectors so that $\mathbf{x}_i^{'}\mathbf{x}_i = 1$, where $\mathbf{x}_i$ is the i'th eigenvector. Plot the time series for these two risk factors. Do they have any obvious economic interpretation? Next, suppose that asset returns are in fact generated by this 2-factor model so that

$$\widetilde{\mathbf{r}} = \mathbf{E} + \mathbf{b}_1 \widetilde{\delta}_1 + \mathbf{b}_2 \widetilde{\delta}_2 + \widetilde{\varepsilon}$$

where (γ_1, γ_2) are risk premia, and $E[\widetilde{\varepsilon}] = E[\widetilde{\delta}] = E[\widetilde{\varepsilon}\widetilde{\delta}^{'}] = 0$. By construction the factors are orthogonal to each other and they can be normalized to have zero mean and unit variance.

(ii) Construct a $(2x2)$ rotation matrix T with the following properties:

(a) the first rotated factor has a positive risk premium; the second pays a zero risk premium.

(b) the rotated factors are orthogonal to the errors $(\widetilde{\varepsilon})$.

(c) the covariance matrix of the rotated factors is diagonal. (Hint: let the first column of T be proportional to $\boldsymbol{\gamma}$ and then construct T as an orthogonal matrix).

3. The returns on a particular mutual fund is generated by the equation

$$R_t = \alpha + \beta R_{mt} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2),$$

where R_t is the excess return on the fund, β is its beta, R_{mt} is the excess return on the market portfolio, and ε_t is the residual in period t.

(i) Suppose that $\alpha = -0.1$ and it is known that $\beta = 1, \sigma = 1.5$. Assuming that the size of the test is 5%, plot the power of the test

$$H_0 : \alpha = 0$$

against the alternative

$$H_1 : \alpha < 0$$

as a function of the number of time periods (n) used in the test.

(ii) For monthly data the parameter values in (i) correspond to a fund that underperforms by 1.2 percent per year. How many months of data (n) do we need to have a 25/50/75/90 percent chance of identifying the fund as an underperformer?