Assignment 1, ECON 214A

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1. You are in an economy with three states ($K = 3$) and three assets ($N = 3$). Consider the following Payoff matrix D:

$$D = \begin{bmatrix} 1 & 4 & 10 \\ 1 & 3 & 8 \\ 1 & 2 & 6 \end{bmatrix};$$

where the $(i;j)$'th element of D gives the payoff in state i of the j'th asset. Suppose the price vector is given by $p = \begin{bmatrix} 1 & 3 & 8 \end{bmatrix}$.

(i) Is the financial market complete?

(ii) Does the price vector permit arbitrage trading strategies?

(iii) Does there exist a set of equilibrium prices ($Q(\omega_j)$)? If so, derive the restrictions that this/these equilibrium price measure(s) must satisfy.

(iv) Characterize the set of attainable consumption processes.

(v) Characterize the budget set of a consumer with endowment $(e_0; e(\omega_j; T)) = (1; 1; 1; 1)$.

(vi) Prove that the expected cost (under Q) of an attainable consumption process is identical for different equilibrium price measures.

(vii) Does the reult in (vi) hold for consumption processes that are not attainable?

Traders in the economy want to price an option written on the second asset with an exercise price of 3. Hence the payoff on this option is $d_{option} = Max[d_2(\omega_j) - 3; 0]$.

(viii) Does this option have the same expected value under the different Q measures? What is the price of the option?

Now suppose that a fourth asset is added to the above list. This asset has a price of 4 and pays off $d_4(\omega_j) = (5; 3; 4)$ in the three states.

(ix) Derive the price of the option using the expanded set of assets.

2. Prove that if a complete price system permits an arbitrage strategy, then the budget sets of all traders coincide with the consumption set (From Dothan, chapter 1).

3. Explain which assumptions are required on agents' expectations in order to establish the existence of equilibria in a two-period model like the one we analyzed in the lectures. In particular answer whether it is necessary that agents have

(i) Rational expectations

(ii) Identical probability beliefs

4. Consider an economy with a set of K discrete states, $s = 1, \ldots, K$. There are also N assets traded at prices of p_k, with payoffs d_{ks}, $k = 1, \ldots, K$. Investors have utility $U(C_s)$, where C_s is consumption in state s, and the utility function has standard properties. Investors probability beliefs are indexed by π_s, $s = 1, \ldots, K$.

(i) The fundamental theorem of risk bearing states that, at the individual's risk-bearing optimum, the expected marginal utility of a dollar will be identical across states:

$$\frac{\pi_1 U'(C_1)}{p_1^*} = \frac{\pi_2 U'(C_2)}{p_2^*} = \ldots = \frac{\pi_s U'(C_s)}{p_s^*}, \quad s = 1, \ldots, K.$$

Here p_s^* is the price of an (Arrow-Debreu) asset that pays out one dollar in state s.

Explain whether this result requires completeness of markets. How does it relate to Pareto optimality of a market equilibrium?

(ii) The risk bearing theorem for asset markets states that, at the risk bearing optimum, individuals adjust their asset holdings until prices become proportional to the expected marginal utilities derived from the consumptions they generate:

$$\frac{\sum_s \pi_s U'(C_s) d_{1s}}{p_1} = \frac{\sum_s \pi_s U'(C_s) d_{2s}}{p_2} = \ldots = \frac{\sum_s \pi_s U'(C_s) d_{Ns}}{p_N}, \quad s = 1, \ldots, K.$$

Notice that p_1 is the price of a complex asset that may have payoffs in several states.

Does the risk bearing theorem for asset markets require market completeness? Will equilibria be pareto optimal in economies where this theorem holds?