PEF 10,374 — Spring 2019

Topics in International Economics

Problem Set 2

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Instructor: Marc-Andreas Muendler
E-mail: marc-andreas.muendler@unisg.ch

1 Properties of the Pareto and Fréchet Distributions

Consider a Pareto distributed variable $\phi \sim \mathcal{P}(\phi_0, \theta)$, where ϕ_0 is called the location parameter and θ the shape parameter. The Pareto distribution function is $F(Z \leq \phi) = 1 - (\phi_0/\phi)^\theta$.

1. Show that the Pareto density function is $\mu(\phi|\phi_0, \theta) = \theta(\phi_0)^\theta/(\phi)^{\theta+1}$.

2. For any $\phi^* > \phi_0$, show that the conditional distribution function is: $F(\phi|\phi \geq \phi^*) = 1 - (\phi^*/\phi)^\theta$, also a Pareto distribution function.

3. Consider a transformed random variable $A(\phi)^B$ with $A, B > 0$. Show that the transformed variable is Pareto distributed with location parameter $A\,(\phi_0)^B$ and shape parameter θ/B.

4. Show that the mean of a Pareto distributed variable ϕ is $\mathbb{E}\,[\phi|\phi_0, \theta] = \theta\phi_0/(\theta - 1)$ if $\theta > 1$.

5. Consider the Fréchet distribution $G(Z \leq z) = \exp\{-Tz^{-\theta}\}$. Show that the Fréchet distribution approaches the Pareto distribution "in the right tail", that is show that

$$\lim_{z\to\infty} G(Z \leq z) = \lim_{z\to\infty} 1 - Tz^{-\theta}.$$

   (*Hint:* Use L'Hôpital's rule and the fact that $\lim_{z\to\infty} z^{-\theta} = 0$ for $\theta > 0$.)

2 Chaney (AER 2008) and Gravity

There are N countries. A country carries a subscript s if it is the source of exports, and a subscript d if it is a destination ($s, d = 1, \ldots, N$). In each country lives a measure of L_d consumers, who inelastically supply one unit of labor and own the shares of domestic firms. Firms choose to enter their respective home market s and any export destination d. There are source-destination iceberg transportation costs τ_{sd} between countries, and there is a source-destination specific fixed cost of entry F_{sd}.

The L_d representative consumers have identical CES preferences over a continuum of varieties with

$$U_d = \left[\sum_{s=1}^{N} \int_{\omega \in \boldsymbol{\Omega}_{sd}} c(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \qquad \text{for } \varepsilon = \sigma > 1.$$

$\boldsymbol{\Omega}$ is the (fixed) measure of available varieties, given a predetermined measure of firms J_s in every country that sells varieties.

Each firm ω's production technology is constant returns to scale but firms from country s differ in productivity ϕ, which they draw from a Pareto distribution $F(\phi) = 1 - (b_s/\phi)^{\theta}$. It will be convenient to call all firms ω with a given productivity level the firms ϕ.

1. Show that consumer demand for a firm ϕ's variety $c_{sd}(\phi)$ is

$$c_{sd}(\phi) = \frac{(p_{sd})^{-\sigma}}{(P_d)^{1-\sigma}} \, y_d L_d$$

for the ideal price index

$$P_d \equiv \left(\int_{\phi \in \boldsymbol{\Omega}_{sd}} p_{sd}^{1-\sigma} \, \mathrm{d}\phi \right)^{\frac{1}{1-\sigma}}.$$

2. Upon entry, each firm maximizes operational profits

$$\pi_{sd}(\phi) = \left(p_{sd} - \tau_{sd} \, \frac{w_s}{\phi} \right) \left(\frac{p_{sd}}{P_d} \right)^{-\sigma} \frac{y_d L_d}{P_d} - F_{sd}.$$

Show that optimal price is a constant markup over unit production cost with

$$p_{sd}(\phi) = \eta \, \frac{\tau_{sd} \, w_s}{\phi}, \qquad \eta = \frac{\sigma}{\sigma - 1}.$$

3. Derive a firm ϕ's total sales $p_{sd}(\phi) q_{sd}(\phi)$ and show that optimal operational profits satisfy $\pi_{sd}(\phi) = p_{sd}(\phi) q_{sd}(\phi)/\sigma$. Using results from Question 1 show that total sales $p_{sd}(\phi) q_{sd}(\phi)$ are Pareto distributed.

4. Show that the least productive firm from country s with a productivity

$$\phi^*_{sd} = \left(\frac{\sigma F_{sd}}{y_d L_d} \right)^{\frac{1}{\sigma-1}} \frac{\eta \tau_{sd} w_s}{P_d}.$$

just breaks even in destination market d. Explain the intuition.

5. Using results from Question 1, derive aggregate exports

$$X_{sd} = \int_{\phi_{sd}^*}^{\infty} p_{sd}(\phi) q_{sd}(\phi) \mu(\phi | \phi_{sd}^*, \theta) \, \mathrm{d}\phi$$

from source country s to destination d, where $\mu(\phi|\phi_{sd}^*, \theta)$ is the conditional Pareto density for firms from s active at d. State X_{sd} as a function of τ_{sd} and ϕ_{sd}^*. Using the result for ϕ_{sd}^* from above, simplify further.

6. Derive the elasticity of aggregate trade with respect to variable trade costs $\partial \log X_{sd}/\partial \log \tau_{sd}$, using the expression of X_{sd} as a function of τ_{sd} and $\phi_{sd}^*(\tau_{sd})$. Interpret the two terms.

7. Derive the elasticity of aggregate trade with respect to fixed trade costs $\partial \log X_{sd}/\partial \log F_{sd}$.

3 Offshoring as a Rybzcynski Effect

There are two industries 1 and 2 and two factors of production: non-offshorable labor N and offshorable labor L. Non-offshorable labor earns a wage s and offshorable labor earns a wage w. Each industry i's production function $Q_i = AF_i(N_i, L_i)$ is homogeneous of degree one. The foreign country's production functions are identical up to a Hicks-neutral productivity parameter: $Q_i = A^* F_i(N_i, L_i)$. Suppose $A^* < A$. Throughout this question, assume that factor-price equalization occurs: $w/w^* = s/s^* = A/A^*$ and that A and A^* are constant.

Introduce one additional type of trade: firms can have L-tasks performed offshore. Onshore and offshore tasks are not perfect substitutes. They are combined through a Cobb-Douglas production function

$$L_i = (L_i^{\text{on}})^{1-\gamma} (L_i^{\text{off}})^{\gamma},$$

where L_i^{off} denotes offshored labor and $\gamma \in (0, 1)$ captures the intensity of onshore labor. When offshored, a foreign unit of labor costs βw^* for the home economy to contract foreign labor at a distance.

To standardize the analysis, consider industry 1 to be relatively intensive in offshorable L-labor and industry 2 intensive in non-offshorable N-labor. Only the offshoring cost parameter β in the model is free to change.

1. Derive optimal inputs L_i^{on} and L_i^{off} from the cost minimization problem for $wL_i^{\text{on}} + \beta w^* L_i^{\text{off}}$ given some $L_i = \bar{L}_i$, and express L_i^{on} as a function of L_i^{off}. What is the elasticity of substitution between L_i^{on} and L_i^{off}: $-\mathrm{d} \ln L_i^{\text{on}}/\mathrm{d} \ln L_i^{\text{off}}$?

2. What is the shadow price $\bar{w}$ of L_i given optimal choices of L_i^{on} and L_i^{off}? [*Hint*: Obtain the Lagrange multiplier from the cost minimization problem.] Express the ratio of shadow prices $\bar{w}/\bar{w}^*$ as a function of the productivity ratio A/A^* given factor-price equalization $w/w^* = A/A^*$. State demand for L_i^{off} as a function of wages, parameters and L_i.

3. Under these functional forms, is there offshoring for $\beta w^* > w$? In other words, is $A^* < A$ a necessary condition for offshoring? Is there two-way offshoring (home contracting from abroad and abroad contracting from home)? Show that your answers hold for any function $L_i = G(L_i^{\text{off}}; L_i^{\text{on}})$ that satisfies the Inada conditions $\lim_{L_i^{\text{off}} \to 0} \partial G(L_i^{\text{off}}; L_i^{\text{on}})/\partial L_i^{\text{off}} = \infty$ and $\lim_{L_i^{\text{off}} \to \infty} \partial G(L_i^{\text{off}}; L_i^{\text{on}})/\partial L_i^{\text{off}} = 0$.

4. Use the first-order conditions for optimal inputs to show that

$$\hat{L}_i^{\text{off}} = \hat{L}_i^{\text{on}} - \hat{\beta} = \hat{L}_i - (1-\gamma)\,\hat{\beta},$$

   where hats over a variable x denote relative changes $\hat{x} \equiv \mathrm{d} \ln x$.

5. Show that, in equilibrium, total offshorable labor supply to the home economy is

$$L = L_1 + L_2 = (L^{\text{on}})^{1-\gamma}(L^{\text{off}})^{\gamma}.$$

   [*Hint*: Use the fact that $L_i^{\text{on}}/L_i^{\text{off}}$ is a constant across both industries for given β.]

6. For simplicity, suppose from now on that offshoring goes only one way, with domestic production using foreign offshore labor but not the reverse (β for offshoring by the domestic economy is finite, but infinite for offshoring by the foreign economy). Factor market clearing then is equivalent to

$$\begin{aligned} a_{L1}Q_1 + a_{L2}Q_2 &= (L^{\text{on}})^{1-\gamma}(L^{\text{off}})^{\gamma} \\ a_{N1}Q_1 + a_{N2}Q_2 &= N \end{aligned}$$

   for the unit labor requirements $a_{Lj} = L_j/Q_j$ and $a_{Nj} = N_j/Q_j$. Show that

$$\begin{aligned} \alpha_{L1}\hat{Q}_1 + (1-\alpha_{L1})\hat{Q}_2 &= (1-\gamma)\hat{L}^{\text{on}} + \gamma \hat{L}^{\text{off}} \\ \alpha_{N1}\hat{Q}_1 + (1-\alpha_{N1})\hat{Q}_2 &= \hat{N} \end{aligned}$$

   for adequately defined α_{L1} and α_{N1}. State α_{L1} and α_{N1}.

7. What does inelastic labor supply and the absence of cross-border migration imply for $\hat{L}^{\text{on}}$ and $\hat{N}$?

8. Use inelastic labor supply and the results from 4 and 5 to show that

$$\begin{pmatrix} \hat{Q}_1 \\ \hat{Q}_2 \end{pmatrix} = -\frac{\gamma}{\alpha_{L1} - \alpha_{N1}} \begin{pmatrix} 1 - \alpha_{N1} \\ -\alpha_{N1} \end{pmatrix} \cdot \hat{\beta}.$$

   Under the assumptions made in the beginning, what are the signs of $\hat{Q}_1$ and $\hat{Q}_2$?

9. Much of the empirical literature on wage inequality and trade uses wage-bill shares in estimation. Define the onshore wage-bill share of non-offshorable labor in industry i as

$$\theta_{Ni}^{\text{on}} = \frac{sN_i}{wL_i^{\text{on}} + sN_i}.$$

   Show that the relative change in the wage-bill share of non-offshorable labor is

$$\hat{\theta}_{Ni}^{\text{on}} = (1 - \theta_{Ni}^{\text{on}})(\hat{s} - \hat{w} + \hat{N}_i - \hat{L}_i^{\text{on}}).$$

10. Suppose factor-price equalization holds. Use the results from 8 and 9 to derive $\hat{\theta}_{N1}^{\text{on}}$ and $\hat{\theta}_{N2}^{\text{on}}$ as functions of θ_{Ni}^{on}, parameters and $\hat{\beta}$. Do the $\hat{\theta}_{Ni}^{\text{on}}$ responses to $\hat{\beta}$ differ in sign? How do their responses to $\hat{\beta}$ differ in magnitude?

4 Translog Cost Functions

Burgess (REStat 1974) has extended Christensen, Jorgenson & Lau's (REStat 1973) single-product translog (transcendental logarithmic) cost function to the case of multiple products (such as products shipped to N different destination markets or made in N different source countries):

$$\begin{aligned} \ln C_j &= \alpha + \sum_{k=1}^{N} \alpha_k \ln Q_j^k + \sum_{\ell=1}^{N} \tau_\ell \ln w_\ell + \sum_{k=1}^{N}\sum_{\ell=1}^{N} \chi_{k\ell} \ln Q_j^k \ln w_\ell \\ &\quad + \frac{1}{2}\sum_{k=1}^{N}\sum_{\ell=1}^{N} \lambda_{k\ell} \ln Q_j^k \ln Q_j^\ell + \frac{1}{2}\sum_{k=1}^{N}\sum_{\ell=1}^{N} \delta_{k\ell} \ln w_k \ln w_\ell, \end{aligned} \tag{1}$$

where the subscript j denotes a firm or an industry, depending on application, Q_j^ℓ is output at or for location ℓ, and w_ℓ is a factor price at or for location ℓ. There are N locations that differentiate the product.

1. Is the cost function (1) separable in individual products for product-level cost functions $c_j^\ell(\cdot)$ so that $C_j(\mathbf{Q}_j; \mathbf{w}) = \sum_\ell c_j^\ell(Q_j^\ell; \mathbf{w})$?

2. For (1) to be homogeneous of degree one in factor prices for any given output vector $\mathbf{Q}_j$, parameters must satisfy certain conditions. What condition does $\sum_{\ell=1}^{N} \tau_\ell$ have to satisfy? What does $\sum_{\ell=1}^{N} \chi_{k\ell}$ have to satisfy for all k? What condition do the sums $\sum_{k=1}^{N} \delta_{k\ell}$, $\sum_{\ell=1}^{N} \delta_{k\ell}$ and $\sum_{k=1}^{N} \sum_{\ell=1}^{N} \delta_{k\ell}$ have to satisfy? By symmetry, we must have $\delta_{k\ell} = \delta_{\ell k}$. How many symmetry restrictions are there for N locations?

Now consider capital K^ℓ a quasi-fixed factor in the short run. Following Brown & Christensen (equation 10.21 of chapter 10 in Berndt & Field 1981: *Modeling and measuring natural resource substitution*), one can augment (1) to a short-run translog multiproduct cost function

$$\begin{aligned}
\ln C_j^V \quad = \quad & \alpha + \sum_{k=1}^{N} \alpha_k \ln Q_j^k + \sum_{\ell=1}^{N} \tau_\ell \ln w_\ell + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \chi_{k\ell} \ln Q_j^k \ln w_\ell \\
& + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \lambda_{k\ell} \ln Q_j^k \ln Q_j^\ell + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \delta_{k\ell} \ln w_k \ln w_\ell \\
& + \sum_{k=1}^{N} \kappa_k \ln K_j^k + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \mu_{k\ell} \ln K_j^k \ln Q_j^\ell \qquad (2) \\
& + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \zeta_{k\ell} \ln K_j^k \ln w_\ell + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \psi_{k\ell} \ln K_j^k \ln K_j^\ell .
\end{aligned}$$

3. What additional condition on $\sum_{\ell=1}^{N} \zeta_{k\ell}$ is now needed for linear homogeneity of (2) in factor prices?

4. Use Shepard's Lemma to derive firm or industry j's demand for factor ℓ from (2).

5. Show that the cost share of factor ℓ in j's total costs C_j^V is

$$\theta_j^\ell \quad = \quad \tau_\ell + \sum_{k=1}^{N} \chi_{k\ell} \ln Q_j^k + \sum_{k=1}^{N} \zeta_{k\ell} \ln K_j^k + \sum_{k=1}^{N} \delta_{k\ell} \ln w_k .$$

6. The constant-output *cross-price elasticity of substitution* between factors ℓ and k is defined as

$$\varepsilon_{\ell k} \equiv \frac{\partial \ln X_j^\ell}{\partial \ln w_k} = w_k \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \right),$$

where X_j^ℓ is factor demand. Show that the second equality follows from Shepard's Lemma. Derive the cross-price elasticity of substitution ($\ell \neq k$ off diagonal) and the own-price elasticity ($\ell = k$ on diagonal) for the translog cost function C_j^V.

7. The partial *Allen-Uzawa elasticity of substitution* between two factors of production ℓ and k is defined as

$$\sigma_{\ell k}^{AU} \equiv C_j \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \frac{\partial C_j}{\partial w_k} \right) = \frac{\varepsilon_{\ell k}}{\theta_j^k},$$

where $\varepsilon_{\ell k}$ is the (constant-output) cross-price elasticity of factor demand and θ_j^k is the share of the kth input in total cost. Show that the second equality follows from Shepard's Lemma. Derive the Allen-Uzawa elasticity on and off the diagonal for the translog cost function C_j^V.

8. *Morishima* elasticities are superior to Allen-Uzawa elasticities. Blackorby & Russel (AER 1989) show that, among other benefits, Morishima elasticities preserve Hicks's notion that the elasticity of substitution between two factors of production should completely characterize the curvature of an isoquant. Allen-Uzawa elasticities fail in this regard when there are more than two inputs. The *Morishima elasticity of substitution* can be derived as a natural generalization of Hicks's two-factor elasticity and is defined as

$$\sigma_{\ell k}^{M} \equiv w_\ell \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_k} \right) - w_\ell \cdot \frac{\partial^2 C_j}{(\partial w_\ell)^2} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \right) = \varepsilon_{k\ell} - \varepsilon_{\ell\ell},$$

where $\varepsilon_{k\ell}$ is the (constant-output) cross-price elasticity of factor demand. Show that the second equality follows from Shepard's Lemma. Derive the Morishima elasticity on and off the diagonal for the translog cost function C_j^V. [*Note*: Morishima elasticities are inherently asymmetric because Hicks's definition requires that only the price w_ℓ in the ratio w_ℓ / w_k vary.]