Economics 247 — Spring 2018

International Trade

Problem Set 1

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Due:	**Tue, April 24, 2018**
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1 Dornbusch-Fischer-Samuelson's Ricardian Model with Unbalanced Trade

Consider a version of the Dornbusch-Fischer-Samuelson model of Ricardian trade with transport costs and a non-zero trade balance. There is a continuum of goods indexed with $z \in [0, 1]$. There are symmetric iceberg transportation cost so κ melts away and $1/(1-\kappa)$ units of a product need to be made for one unit to arrive abroad.

Consumers have homothetic preferences with consumption basket $C_d \equiv \exp\{\int_0^1 \ln c_d(z)\mathrm{d}z\}$. Using a result from question 3, demand for a product z is $c(z) = PC/p(z)$ with the ideal price index $P = \exp\{\int_0^1 \ln p(z)\mathrm{d}z\}$.

Labor is only factor of production and makes a product z under unit labor requirements $a(z)$. Define the Home country's comparative advantage in industry z with $A(z) \equiv a^*(z)/a(z)$. Assume without loss of generality that z strictly indexes the industries with the Home's strongest comparative advantage so that $A'(z) < 0$.

1. Using the condition $w\,a(z) \leq w^*\,a^*(z)/(1-\kappa)$ for home production, determine the cutoff industry z^H up to which the home country produces. Similarly, using the condition $w^*\,a^*(z) \leq w\,a(z)/(1-\kappa)$ for foreign production, determine the cutoff industry z^F up to which the foreign country produces. Show that $z^H > z^F$ for $\kappa > 0$ and $A'(z) < 0$.

2. To simplify exposition, consider the functional form $A(z) = \exp\{1 - 2z\}$. Show that the size of the nontraded sector $z^H - z^F$ can then be expressed as $z^H - z^F = -\log(1-\kappa) > 0$.

3. In equilibrium, global consumption expenditure must equal global income so that $PC + P^*C^* = wL + w^*L^*$ ("market clearing"). Home income equals global expenditure on home produced goods so $wL = z^H PC + z^F P^*C^*$, and a similar expression applies to the foreign country. Define the home trade balance as $TB = wL - PC = -TB^* \neq 0$, that is the

excess output over absorption. Make good 1 the numeraire, a foreign produced good, so that $w^* = p^*(1)/a^*(1) = 1/a^*(1)$. Show that the global "market clearing" condition and $TB = wL - PC \neq 0$ imply

$$\frac{w}{w^*} = \left(\frac{\log(1-\kappa)TB}{L^*/a^*(1)} + z^F \right) \frac{L^*/L}{1 + \log(1-\kappa) - z^F} \equiv B(z^F).$$

4. Using the cutoff for foreign production $w^* \, a^*(z^F) = w \, a(z^F)/(1-\kappa)$, place conditions on TB so that that this relationship and $B(z^F)$ above result in a unique equilibrium. (*Hint:* Establish monotonicity and limits. Start with $TB = 0$, then generalize.)

5. How does the equilibrium with a non-zero trade balance differ from that derived under a zero trade balance? How does an increase in the home trade balance TB affect the location of industries? How does the increase affect the size of the nontraded sector under $A(z) = \exp\{1 - 2z\}$ and in general? How does the increase affect welfare in the home country?

2 Heckscher-Ohlin Model with Two Countries, Two Industries and Two Factors

There are two industries 1 and 2 and two factors of production: capital K and labor L. Capital earns a rental rate r and labor a wage w. Each industry i's production function $Q_i = AF_i(K_i, L_i)$ is homogeneous of degree one. The foreign country's production functions are identical up to a Hicks-neutral productivity parameter: $Q_i = A^* F_i(K_i, L_i)$.

1. A function $f(x, y)$ is homogeneous of degree α if $f(\lambda x, \lambda y) = \lambda^\alpha f(x, y)$ for any $\lambda > 0$. Differentiate the production function $Q_i = AF_i(K_i, L_i)$ with respect to L_i. Is the marginal product $A \, \partial F_i(K_i, L_i)/\partial L_i$ homogeneous, if so of what degree? Use your result to show that $A \, \partial F_i(K_i/L_i, 1)/\partial L_i = A \, \partial F_i(K_i, L_i)/\partial L_i$.

From now on, define $f_i(k_i) \equiv F_i(K_i, L_i)/L_i = F_i(K_i/L_i, 1)$, where $k_i \equiv K_i/L_i$ is industry i's capital-labor ratio.

2. State the input rules, by which each factor's income r and w equals the marginal revenue product in each local industry. Derive the wage-rental ratio $\omega \equiv w/r$ as a function of k_i and show that ω is not a function of A (or A^*). Derive the total differential $\mathrm{d}\omega/\mathrm{d}k_i$ as a function of k_i for each industry and show that it is not a function of A (or A^*).

3. Use the input rule for labor in each industry to show the relationship between the product price ratio $p \equiv P_1/P_2$ and the two industries' marginal products in terms of labor. How does the slope of the production possibility frontier relate to the two industries' marginal products in terms of labor? How does the slope of the production possibility frontier relate to the two industries' marginal products in terms of capital? Show that the slope of the production possibility frontier is not a function of A (or A^*).

4. Show that
$$\frac{\mathrm{d}p}{\mathrm{d}\omega} = \frac{\omega\,(k_2 - k_1)}{(\omega + k_1)(\omega + k_2)}\,\frac{p}{\omega}.$$
Does this relationship allow you to state the Stolper-Samuelson theorem?

5. Suppose international product markets are fully integrated but that capital and labor are completely immobile across borders. So both countries face the same product price ratio $p \equiv P_1/P_2$. Using the above findings, how does an industry i's capital-labor ratio in the home country k_i differ from the same industry's capital-labor ratio in the foreign country k_i^*? Using the input rules, show that the relative wages w/w^* and relative rental rates r/r^* are equal in free-trade equilibrium. What does the result imply about factor price equalization (FPE)?

From now on, assume Cobb-Douglas production functions $q_i = Af_i(k_i) = (k_i)^{\alpha_i}$ and $q_i^* = A^* f_i(k_i^*) = (k_i^*)^{\alpha_i}$, and assume that industry 2 is more capital intensive with $\alpha_2 > \alpha_1$.

6. Rederive the wage-rental ratio $\omega \equiv w/r$ as a function of k_i and show that $k_i = [\alpha_i/(1-\alpha_i)]\omega$. Can there be a factor-intensity reversal? For a Cobb-Douglas production function, does it matter whether A is Hicks-neutral or capital augmenting (such as in $q_i = (Ak_i)^{\alpha_i}$)?

7. Show that
$$p = \frac{(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\,\omega^{(\alpha_2-\alpha_1)}.$$
Based on this result, state the Stolper-Samuelson theorem in its weak form for the Cobb-Douglas production function.

8. Use the factor market clearing conditions $L_1 + L_2 = \bar{L}$ and $K_1 + K_2 = \bar{K}$ together with production to derive as an intermediate step the relative supply relationship as a function of relative price p:
$$\frac{Q_1}{Q_2} = \frac{\kappa_1}{1-\kappa_1}\,\frac{\alpha_2}{\alpha_1}\,\frac{1}{p},$$

where $\kappa_1 \equiv K_1/\bar{K} = 1 - \kappa_2$. Finally, use the above p-ω-relationship and the equilibrium level of κ_1 (as a function of p) to establish

$$\frac{Q_1}{Q_2} = \frac{1 - \left(\frac{\alpha_1}{\alpha_2}\,\frac{1-\alpha_2}{1-\alpha_1}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}} \left(\frac{1-\alpha_2}{1-\alpha_1}\,\frac{1}{p}\right)^{-\frac{1}{\alpha_2-\alpha_1}}\,\frac{\bar{L}}{\bar{K}}}{\left(\frac{\alpha_1}{\alpha_2}\,\frac{1-\alpha_2}{1-\alpha_1}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}} \left(\frac{1-\alpha_2}{1-\alpha_1}\,\frac{1}{p}\right)^{-\frac{1}{\alpha_2-\alpha_1}}\,\frac{\bar{L}}{\bar{K}} - 1} \cdot \frac{1-\alpha_2}{1-\alpha_1}\,\frac{1}{p}.$$

Based on this result, and under the assumption of homothetic preferences, derive the Heckscher-Ohlin theorem for the Cobb-Douglas production function. What happens if the numerator or the denominator in the first term on the right-hand side vanish?

3 Demand Systems and Microfoundations of the Price Index

For the following four demand systems, find the minimum expenditure P that just allows for one unit of the consumption basket $C = 1$. In this formulation, the micro-foundation of the price index P is the value function of the expenditure minimization problem.

1. For the CES Consumption Index $C \equiv \left(\int_0^1 \alpha(z)^{1/\theta} c(z)^{\frac{\theta-1}{\theta}} \mathrm{d}z\right)^{\frac{\theta}{\theta-1}}$, minimize expenditure $\int_0^1 p(z)c(z)\mathrm{d}z$ for all $\{c(z)\}_0^1$ such that $C \geq 1$, state the first-order conditions, derive Hicksian demands and show that $P = \left(\int_0^1 \alpha(z)p(z)^{1-\theta}\mathrm{d}z\right)^{\frac{1}{1-\theta}}$.

2. For the Log Consumption Index $C = \left\{\int_0^1 \ln c(z)\mathrm{d}z\right\}$, minimize expenditure $\int_0^1 p(z)c(z)\mathrm{d}z$ for all $\{c(z)\}_{z\in[0,1]}$ such that $C \geq 0$ and show that $P = \exp\left[\int_0^1 \ln p(z)\mathrm{d}z\right]$.

3. For the CES index in Tradables and Nontradables $C \equiv \left[\gamma^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_S^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$, minimize expenditure $C_T + p\,C_S$ for $\{C_S, C_T\}$ such that $C \geq 1$ and show that $P = \left[\gamma + (1-\gamma)p^{1-\theta}\right]^{1/(1-\theta)}$.

4. For Cobb-Douglas utility in Home and Foreign goods $C \equiv \frac{(C_h)^\gamma (C_f)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$, minimize expenditure $p_h\,C_h + p_f\,C_f$ for $\{C_h, C_f\}$ such that $C \geq 1$ and show that $P = (p_h)^\gamma (p_f)^{1-\gamma}$.

5. Compute the limits of the CES Consumption Index $C \equiv [aC_1^\rho + (1-a)C_2^\rho]^{1/\rho}$ as $\rho \to -\infty$ and $\rho \to 0$. (*Hint:* Apply L'Hôpital's rule to the log CES index.) Compute the elasticity of substitution between C_1 and C_2. What is the elasticity of substitution for $\rho \in \{-\infty, 0, 1\}$?