Economics 246 — Fall 2004

International Macroeconomics

# Problem Set 1

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**Due:** **Mon, October 18**
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## 1 Productivity Shocks with Initially Unbalanced Current Accounts

There are two periods and two countries. Home produces with $Y = AF(K)$ and Foreign with $Y^* = A^*F(K^*)$, where $A$ and $A^*$ are productivity parameters. Both countries' representative agents have the same period utility so that $U_1 = u(C_1) + \beta u(C_2)$ and $U_1^* = u(C_1^*) + \beta u(C_2^*)$. Assume period utility $u(\cdot)$ to be *isoelastic*. International financial market clearing $S + S^* = I + I^*$ (or $CA_1 = -CA_1^*$) determines the world interest rate $r$.

1. Use the intertemporal optimality conditions for production to show that an anticipated productivity shock $dA_2/A_2$ to production in Home changes equilibrium investment and the interest rate in the following way

$$r\frac{\mathrm{d}A_2}{A_2} + \left(\frac{\partial I_1}{\partial r}\right)^{-1} \mathrm{d}I_1 - \mathrm{d}r = 0.$$

   Derive the according equation for Foreign and $dA_2^*/A_2^*$.

2. Use the intertemporal optimality conditions for consumption, along with your results in 1, to show that an anticipated productivity shock $dA_2/A_2$ to production in Home changes the equilibrium current account level and the interest rate in the following way

$$D\,\frac{\mathrm{d}A_2}{A_2} - M\,\mathrm{d}r + V\,\mathrm{d}CA_1 = 0,$$

   for some functions $V, D$, and $M$. Derive the according equation for Foreign and $dA_2^*/A_2^*$ (with $V^*, D^*, M^*$ and using $CA_1 = -CA_1^*$). Derive the signs of $V, V^*, D$ and $D^*$.

3. Show that anticipated productivity shocks to Home and Foreign have the following effect on the equilibrium interest rate and the Home current account:

$$\begin{aligned}
\mathrm{d}r &= \frac{1}{\frac{M}{V} + \frac{M^*}{V^*}}\left(\frac{D}{V}\frac{\mathrm{d}A_2}{A_2} + \frac{D^*}{V^*}\frac{\mathrm{d}A_2^*}{A_2^*}\right) \\
\mathrm{d}CA_1 &= -\frac{1}{\frac{V}{M} + \frac{V^*}{M^*}}\left(\frac{D}{M}\frac{\mathrm{d}A_2}{A_2} - \frac{D^*}{M^*}\frac{\mathrm{d}A_2^*}{A_2^*}\right).
\end{aligned}$$

4. Suppose Home runs a current account surplus $CA_1 > 0$ during period 1. How does an anticipated increase in Home productivity $A_2$ during period 2 affect the current accounts in period 1? Is the effect unambiguous? Why or why not?

5. Suppose again Home runs a current account surplus $CA_1 > 0$ during period 1. How does an anticipated increase in Foreign productivity $A_2^*$ during period 2 affect the current accounts in period 1? Is the effect unambiguous? Why or why not?

6. Consider anticipated and equal proportional increases in Home and Foreign productivity $dA_2/A_2 = dA_2^*/A_2^*$. How does this change affect worldwide investment $I_1 + I_1^*$ during period 1? Does the answer depend on the elasticity of intertemporal substitution? Is the effect unambiguous? Why or why not?

# 2 Exponential Period Utility

There are two periods. A country's representative household has the exponential period utility function

$$u(C) = -\gamma \exp(-C/\gamma)$$

with $\gamma \in (0, \infty)$ and maximizes lifetime utility $U_1 = u(C_1) + \beta u(C_2)$ subject to

$$C_1 + RC_2 = Y_1 + RY_2 \equiv W,$$

where $R \equiv 1/(1+r)$ is the price of tomorrow's consumption in terms of today's consumption and $W$ is initial wealth. The value of $W$ depends on $R$.

1. Derive the Euler equation and solve it for $C_2$ as a function of $C_1$, $R$ and $\beta$.

2. What is the optimal level of $C_1$ considering $W$, $R$ and $\beta$ as given?

3. Differentiate this consumption function of $C_1$ with respect to $R$ (differentiate $W$ with respect to $R$ too) and show that

$$\frac{dC_1}{dR} = -\frac{C_1}{1+R} + \frac{Y_2}{1+R} + \frac{\gamma}{1+R}\left(1 - \ln(\beta/R)\right)$$

4. Derive the intertemporal elasticity of substitution of the exponential period utility $(-u'(C)/Cu''(C))$.

5. Use this result to show that the derivative $dC_1/dR$ in part 3 can be expressed as

$$\frac{dC_1}{dR} = \frac{\sigma(C_2)\, C_2}{1+R} - \frac{C_2}{1+R} + \frac{Y_2}{1+R}.$$

   Interpret the three additive terms in this derivative.

# 3 Stochastic Current Account Model

There are infinitely many periods. A country's representative household has the linear-quadratic period utility function

$$u(C) = C - \frac{a_0}{2}C^2$$

with $a_0 \in (0, \infty)$ and maximizes lifetime utility

$$U_t = \mathbb{E}_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} u\left(C_s\right) \right]$$

subject to

$$CA_s = B_{s+1} - B_s = rB_s + \tilde{Z}_s - C_s \quad \forall s \geq t$$

where $R \equiv 1/(1+r) = \beta$ and $\tilde{Z}_s (\equiv \tilde{Y}_s - \tilde{G}_s - \tilde{I}_s)$ is *random* net output.

1. Derive the stochastic Euler equations and show that $C_t$ satisfies

$$C_t = rR \left( (1+r)B_t + \sum_{s=t}^{\infty} R^{s-t} \mathbb{E}_t[\tilde{Z}_s] \right)$$

2. Show that then $CA_t \equiv B_{t+1} - B_t = \tilde{Z}_t - \mathbb{E}_t[\hat{\tilde{Z}}_t]$, where the hat denotes the permanent level of the variable. The permanent level $\hat{X}$ of a random variable $\tilde{X}$ is defined as $\sum_{s=t}^{\infty} R^{s-t} \hat{\tilde{X}} \equiv \mathbb{E}_t \left[ \sum_{s=t}^{\infty} R^{s-t} \tilde{X}_s \right]$.

3. Define $\Delta\tilde{Z}_s \equiv \tilde{Z}_s - \tilde{Z}_{s-1}$ and show that the current account can be rewritten as $CA_t = -\sum_{s=t+1}^{\infty} R^{s-t} \mathbb{E}_t \left[ \Delta\tilde{Z}_s \right]$.

4. Use $\Delta\tilde{Z}_t \equiv \tilde{Z}_t - \tilde{Z}_{t-1}$ to show that the quantity

$$CA_t - (1+r)CA_{t-1} - \Delta\tilde{Z}_t$$

is stochastically independent of $CA_s$ and $\Delta\tilde{Z}_s$ for all $s < t$. How is this finding related to Hall's (1978) result that consumption follows a random walk?

# 4 Current Account and Terms of Trade

In a small open economy, the representative individual maximizes the lifetime utility function

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(X_s^{\gamma} M_s^{1-\gamma}\right)^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where $X$ is consumption of an exported good and $M$ consumption of an imported good. The country completely specializes in production of the export good. The endowment of this good is constant at $Y$. The representative individual faces the fixed world interest rate $r = (1-\beta)/\beta$ in terms of the real consumption index $C = X^{\gamma} M^{1-\gamma}$ (so a loan of 1 real consumption unit today returns $1 + r$ real consumption units tomorrow). There is no investment or government spending.

1. Let $p$ bet the price of the export goods in terms of the import good. So, a rise in $p$ is an improvement in the terms of trade. Show that the consumption-based price index $P$ in terms of imports is

$$P = p^{\gamma}/\gamma^{\gamma}(1-\gamma)^{1-\gamma}.$$

2. Show that the home country's current account identity is

$$B_{t+1} - B_t = rB_t + \frac{p_t(Y - X_t)}{P_t} - \frac{M_t}{P_t}.$$

   What is the corresponding intertemporal budget constraint for the representative consumer?

3. Derive the first-order conditions of the consumer's problem. (Hint: Reformulate the utility function and budget constraint in terms of real consumption $C$.) What are the optimal paths for $X$ and $M$?

4. Suppose initial expectations are that $p$ remains constant over time. There is an unexpected *temporary* fall in the terms of trade from $p$ to $p' < p$. What is the effect on the current account $CA_t = B_{t+1} - B_t$ from part 2?

5. Now suppose foreign net wealth $B$ is indexed to the import good $M$ rather than to real consumption. Accordingly, let $r$ denote the own-rate of interest in imports but assume again that $r = (1-\beta)/\beta$. How does a *temporary* drop in the terms of trade from $p$ to $p' < p$ affect the current account now? How do you explain differences, if any, to part 4?