Econ 202a Spring 2000
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Problem Set 1 (Part 2): Suggested Solutions

1 Question 5

In our stylized economy, the logarithm of aggregate demand is implicitly given by

$$p_t = m_t - a y_t + \tilde{u}_t, \tag{1}$$

and the logarithm of aggregate supply by

$$y_t = b\,(p_t - \mathbb{E}[p_t]) + k + \tilde{v}_t. \tag{2}$$

Both $\tilde{u}_t$ and $\tilde{v}_t$ are random-variables with zero mean $\mathbb{E}[\tilde{u}_t] = \mathbb{E}[\tilde{v}_t] = 0$.

1.1 [5a] Aggregate supply

The aggregate supply function (2) takes the form of a stochastic Lucas supply curve

$$y_t - k = b\,(p_t - \mathbb{E}[p_t]) + \tilde{v}_t.$$

The deviation of log output from its mean is a linear function of the deviation of the log price level from its mean. Such an aggregate supply function arises in general equilibrium when all prices are flexible and individuals form rational expectations. The reason is the following: When each individual observes a high price or wage for his or her own production goods or labor supply, it rationally believes that this can be due to a real shock (with some correctly derived probability) or a nominal shock (with some correctly derived probability) or both to certain degrees (with correctly derived probabilities). If the individual knew for sure that only a nominal shock had hit the economy, she would not change her production and consumption decision at all. But there is some positive probability that a real shock caused the prices to deviate from their mean value. Therefore, each individual chooses to produce and consume more when prices are above their mean, and to produce and consume less when prices fall below their mean. After all, the price deviation could stem from a real shock with a certain likelihood. Since this reasoning applies to each individual in the same manner, aggregate output will increase when the aggregate price level happens to be above its mean, and fall when the aggregate price level happens to be below the mean.

1.2 [5b] Equilibrium price level

Since (1) and (2) must both be satisfied in equilibrium, we can plug (2) into (1) and simplify (by bringing all terms that contain a p_t over to one side). This gives us

$$p_t = \frac{1}{1+ab}\left(m_t + ab\mathbb{E}[p_t] - ak - a\tilde{v}_t + \tilde{u}_t\right). \tag{3}$$

1.3 [5c] Expected equilibrium price level

All individuals form rational expectations. That is, they form expectations on the basis of what they know about the economy. In particular, they know that equilibrium price will be as above, (3). (As a rule: Individuals with rational expectations always know just as much as the economist who builds the model. And the economist who builds the model does not know what the realizations of the random variables will be once the model starts to work—just as the individual economic agent won't know.) So, when these individuals form their expectations about the aggregate price level, they simply take expectations of (3). This yields

$$\mathbb{E}[p_t] = \frac{1}{1+ab}\left(ab\mathbb{E}[m_t] + ab\mathbb{E}[p_t] - ak - \mathbb{E}[a\tilde{v}_t + \tilde{u}_t]\right).$$

Since $\mathbb{E}[a\tilde{v}_t + \tilde{u}_t] = a\mathbb{E}[\tilde{v}_t] + \mathbb{E}[\tilde{u}_t] = 0$, the expectations simplify to

$$\mathbb{E}[p_t] = \mathbb{E}[m_t] - ak. \tag{4}$$

1.4 [5d] Expected equilibrium output

From the supply side of our economy (2), we know that $y_t - k = b(p_t - \mathbb{E}[p_t]) + \tilde{v}_t$. The equilibrium price deviation from its mean, $p_t - \mathbb{E}[p_t]$, is known to us now. We have (3) and (4) from 5c) and 5d). Subtracting one from the other and collecting terms yields

$$p_t - \mathbb{E}[p_t] = \frac{1}{1+ab}\left(m_t - \mathbb{E}[p_t] - ak - a\tilde{v}_t + \tilde{u}_t\right).$$

Since $\mathbb{E}[p_t] = \mathbb{E}[m_t] - ak$, we find

$$p_t - \mathbb{E}[p_t] = \frac{1}{1+ab}\left(m_t - \mathbb{E}[m_t] - a\tilde{v}_t + \tilde{u}_t\right).$$

Finally, we can plug this term into the Lucas supply function (just as all the rational individuals do at this instant), and, after collecting terms, we obtain

$$y_t - k = \frac{b}{1+ab}\left(m_t - \mathbb{E}[m_t]\right) + \frac{1}{1+ab}\left(\tilde{v}_t + b\tilde{u}_t\right). \tag{5}$$

What do we learn from this? (As opposed to all the rational individuals who must already know this from daily experience in the wild but systematic economy, the economists still have to pretend they are surprised when they learn about the equilibrium. Otherwise no other economist listens.)

- Systematic monetary policy will not affect output. What matters for aggregate output are deviations of the realized monetary supply from the expected money supply. Whenever a central bank follows a rule (be it as weird as it may, e.g. $m_t = \sin(t)k^t$, just to create some fluctuations), all individuals will anticipate it and the policy won't matter. In the case of the weird example, $\mathbb{E}[m_t] = \mathbb{E}[\sin(t)k^t] = \sin(t)k^t$. Hence, $m_t - \mathbb{E}[m_t] = 0$, and the rule is totally irrelevant.

- Of course, pure random shocks that the central bankers create will matter. If the central bankers were to have a random number generator run before their meetings and if they were to use the generated random number as their money supply, money would matter. However, this policy doesn't do anything else but add more noise and create a higher variance in output. Thus, it doesn't do any good. With this random policy, there is not only one $\tilde{u}_t$ that enters aggregate demand, there is an additional $\tilde{u}_t^{extra}$ because the central bankers choose $m_t = \tilde{u}_t^{extra}$. The variance of aggregate demand increases (because the variances of $\tilde{u}_t$ and $\tilde{u}_t^{extra}$ add up). As a result, the variance of aggregate output will increase, too.

- (Beyond question) What if the central bankers just became a little systematic? Say, they let a random number generator run that spits out real numbers between minus one and one with a uniform distribution ($f(\tilde{u}_t^{extra}) = \frac{1}{2}$). They agree to accept random numbers only when they are positive, and never when they are negative. In the case of a negative shock, they choose $m_t = 0$. What will happen? Well, the public will believe that the central bank runs a random number generator with a uniform distribution between minus one and one only for a couple of days. Then they will wonder why no negative shocks occur. And as soon as someone finds out about the new rule, they will correctly form new expectations. The mean shock under the new rule is

$$\begin{aligned}\mathbb{E}[m_t] &= \frac{1}{2}\mathbb{E}[m_t|\tilde{u}_t^{extra} < 0] + \frac{1}{2}\mathbb{E}[m_t|\tilde{u}_t^{extra} \geq 0] \\ &= \frac{1}{2}\mathbb{E}[0|\tilde{u}_t^{extra} < 0] + \frac{1}{2}\mathbb{E}[\tilde{u}_t^{extra}|\tilde{u}_t^{extra} \geq 0] = \frac{1}{4}.\end{aligned}$$

  By correctly setting their expectations to $\mathbb{E}[m_t] = \frac{1}{4}$, the rational individuals take the systematic part out of the policy, and the mean deviation $\mathbb{E}[m_t - \mathbb{E}(m_t)] = \mathbb{E}[m_t - \frac{1}{4}]$ is zero again. The systematic part of the policy does not matter, only the unpleasant noise.

- (Beyond beyond question) There is one type of systematic policy, however, that is particularly interesting. It is only possible when we believe that the central bank has superior knowledge. Whereas no individual in the economy can ever get to know the realization of the nominal shock $\tilde{u}_t$, suppose the central bank knows it just a second before it hits. Then a particularly brilliant policy would be to choose $m_t = -\tilde{u}_t$ each period, exactly when the shock hits. With this policy, the central bank would precisely offset any nominal shock whenever it occurs. Solving the system, all individuals soon find out that this brilliant policy only leaves them with the real shock because prices are now solely determined by demand and supply. In particular, aggregate demand decays to $p_t = -ay_t$ (and $\mathbb{E}[p_t] = -a\mathbb{E}[y_t]$), and expected output becomes $\mathbb{E}[y_t] = k$, using (2). Hence, equilibrium output will be $y_t = k + \frac{1}{1+ab}\tilde{v}_t$ now. There are still some fluctuations in output, but the variance of output must have gone down because now only one, the real shock, hits, and not two shocks. The central bank has successfully stabilized the economy a little more—with the help of all individuals who correctly anticipate the systematic monetary policy whenever they make their decisions. So, for this policy to work we need not only superior information of the central bank, but also that all individuals believe that the central bank has superior information and that they believe the central bank will be willing and able to use this superior information correctly. As soon as the central bank tries to systematically deviate from the policy, individuals will rethink their expectations, too.

2 Question 6

Now let aggregate supply depend on

$$y_t = b\,(p_t - w_t) + k + \tilde{v}_t, \tag{6}$$

where w_t is meant to be the log of nominal wages and given as

$$w_t = c\mathbb{E}[p_t] + (1-c)p_{t-1} \qquad c \in (0,1). \tag{7}$$

2.1 [6a] Wage stickyness

Aggregate supply: Taken alone, the aggregate supply function (6) still resembles Lucas supply somewhat. In fact, we can tell a story of microfoundations for exactly half of the relevant cases. Instead of a deviation in log price levels, we now find a term in aggregate supply that is the inverse of the real wage, written in logs: $p_t - w_t = \ln(P_t/W_t) = -\ln(W_t/P_t)$. Whenever the real wage happens to be set too high (that is, higher than in competitive equilibrium), unemployment will occur. The reason is that some workers who would still be willing to work at the going real wage can't find employment. Their marginal product is too low for firms to employ them at this high wage rate. In order to take this microeconomic story to the macro level, b needs to be positive. Then output will fall when the real wage gets too high.

On the other hand, if the real wage drops below the market clearing wage, there will be excess demand for labor. A microeconomist would argue that this will also cause too little employment because some workers will no longer be willing to offer their labor on the market. Output will fall as well (as a microeconomic principle has it, the shorter side of the market rules). However, a macroeconomist might prefer to keep her parameter b positive.

She would argue that the economy can overheat at times, especially when nominal wage adjustment lags behind price adjustment.

Wage Setting: In the wage setting scheme (7), rational expectations still matter. In fact, for $c = 1$ we would be right back in the case of question 5. But now the nominal wage rate is not set equal to price expectations. The wage rate is rather an average between the past price level and the expected price level for this period. The wage setting scheme is adaptive. A possible story underlying this setup is that wages are set before the random shocks hit the economy. Labor contracts typically do not automatically adjust the wage rate to current inflation. Wages are fixed in advance. In a pure rational expectations world ($c = 1$), wage contracts would specify wages to equal $\mathbb{E}[p_t]$. But if expectations are at least partly adaptive ($c < 1$), workers and firms base their decision also on past observations.

2.2 [6b] Equilibrium price and expected equilibrium price

The new aggregate supply-side of the economy can be written in one equation. Plugging (7) into (6) yields

$$y_t = b\,(p_t - c\mathbb{E}[p_t] + (1-c)p_{t-1}) + k + \tilde{v}_t. \tag{8}$$

The demand side is still determined by the old known from (1)

$$p_t = m_t - ay_t + \tilde{u}_t.$$

Plugging the former into the latter and collecting terms we find

$$p_t = \frac{1}{1+ab}\left(m_t + abc\mathbb{E}[p_t] + ab(1-c)p_{t-1} - ak - a\tilde{v}_t + \tilde{u}_t\right). \tag{9}$$

The only news compared to the equilibrium price before, (3), is the adaptive price term in (9). It would vanish for $c = 1$.

The *expected* equilibrium price is a little more subtle to derive now. All individuals will base their decisions on what they could find out about the economy before time t. In particular, they will use all information that they can squeeze out of past variables when forming their rational expectations. That is, they first take expectations of the equilibrium price p_t conditional on p_{t-1}

$$\mathbb{E}[p_t|p_{t-1}] = \frac{1}{1+ab}\Big(\mathbb{E}[m_t|p_{t-1}] + abc\mathbb{E}[p_t|p_{t-1}] \\ +ab(1-c)p_{t-1} - ak - a\mathbb{E}[\tilde{v}_t|p_{t-1}] + \mathbb{E}[\tilde{u}_t|p_{t-1}]\Big), \quad (10)$$

and observe that current shocks must be uncorrelated with any past variable, and p_{t-1} in particular, so that $\mathbb{E}[\tilde{v}_t|p_{t-1}] = \mathbb{E}[\tilde{u}_t|p_{t-1}] = 0$. But then they start to find conditional expectations uninteresting and begin to take expectations of all the conditional expectations. That is, they are so rational that they remember the law of iterated expectations from their last statistics class: $\mathbb{E}[X_t] = \mathbb{E}_y[\mathbb{E}[X_t|y]]$. With the law of iterated expectations, they find $\mathbb{E}[p_t] = \mathbb{E}[\mathbb{E}[p_t|p_{t-1}]]$ and (10) becomes

$$\mathbb{E}[p_t] = \frac{1}{1+ab}\Big(\mathbb{E}[m_t] + abc\mathbb{E}[p_t] + ab(1-c)p_{t-1} - ak\Big).$$

Solving out for $\mathbb{E}[p_t]$ yields

$$\mathbb{E}[p_t] = \frac{1}{1+ab(1-c)}\Big(\mathbb{E}[m_t] + ab(1-c)p_{t-1} - ak\Big). \quad (11)$$

2.3 [6c] Equilibrium output

Finally, we are (as much as our rational individuals) interested in solving for equilibrium output. Let's consider the real wage first, or even better (the log of) its inverse, $p_t - w_t = p_t - c\mathbb{E}[p_t] - (1-c)p_{t-1}$, for this term enters aggregate supply. We know $\mathbb{E}[p_t]$ from (11) and p_t from (9). Plugging (11) for $\mathbb{E}[p_t]$ into (9) and simplifying, we find the following difference equation in prices

$$p_t = \frac{ab(1-c)}{(1+ab(1-c))}p_{t-1} + \frac{1}{(1+ab)(1+ab(1-c))} \times \\ \times\Big((1+ab(1-c))m_t + abc\mathbb{E}[m_t] - a(1+ab)k \\ +(1+ab(1-c))\tilde{u}_t - a(1+ab(1-c))\tilde{v}_t\Big). \quad (12)$$

This first-order difference equation will behave nicely if $\frac{ab(1-c)}{(1+ab(1-c))}$ is less than one in absolute value. Otherwise, prices will explode. Thus, if the product ab is positive, for example, our economy won't blow up. The rest of the variables plays the role of a huge forcing term. (Using lag operators, we could show that any current price level p_t is an infinite weighted sum of all past realizations of this huge forcing term.)

But we are actually after the real wage, or its inverse. Using this difference equation and $\mathbb{E}[p_t]$ from (11) again, we find it. The inverse real wage (in logs) is

$$\begin{aligned} p_t - w_t &= -\frac{1-c}{(1+ab(1-c))}p_{t-1} + \frac{1}{(1+ab)(1+ab(1-c))} \times \\ &\quad \times \Big((1+ab(1-c))m_t - c\mathbb{E}[m_t] - a(1+ab)(1-c)k \\ &\quad +(1+ab(1-c))\tilde{u}_t - a(1+ab(1-c))\tilde{v}_t \Big). \end{aligned} \tag{13}$$

Finally, plugging the real wage into aggregate supply (6), we obtain what we are looking for

$$\begin{aligned} y_t &= b\,(p_t - w_t) + k + \tilde{v}_t \\ &= -\frac{b(1-c)}{(1+ab(1-c))}p_{t-1} + \frac{1}{(1+ab(1-c))}k \\ &\quad + \frac{b}{(1+ab)(1+ab(1-c))}\Big((1+ab(1-c))m_t - c\mathbb{E}[m_t] \Big) \\ &\quad + \frac{1}{1+ab}\Big(b\tilde{u}_t + \tilde{v}_t \Big), \end{aligned} \tag{14}$$

equilibrium output.

Hence, output does not depend on the deviation of money supply from its mean in a linear way. Therefore, systematic monetary policy does have an effect on output in this model.

Is Rational Expectations the only assumption that we need for the result in 5c)? As the result above shows, the absence of stickyness in prices is an implicit requirement for the result in 5c).

Is Rational Expectations the only assumption that we need for the result in 6c)? Note that we needed rational expectations as a solution concept for the entire derivations in question 6. Although the wage setting process is not purely based on rational expectations, all agents take the systematic error in their wage setting institutions into account when they make their decisions. However, rational expectations are not the only assumption that we made along the way in the derivations for question 6, either. As argued in the answer to question 6a), there are no justifiable microfoundations for the aggregate supply function (6). It is a typical Keynesian (ad hoc) assumption to make the parameter b positive and constant. As the *Lucas critique* states, for a convincing model we need to find a functional form for b that is in line with rational expectations and sensible microfoundations. Systematic monetary policy may in fact change the equilibrium level of b. In turn, such an endogenous change of b also affects the functional relationship between the deviation of money supply from its mean and aggregate output, as can be seen from (14). Monetary policy would still cause deviations of the real wage from the market clearing level, so that money would not become neutral. But deviations of the real wage would always cause output to contract, and never to expand.