# Market Access: Enter and Extract, or Merge and Match?

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**Abstract**

Why have so many firms pursued cross-border mergers during the past decade, whereas competitive entry was predominant in the decades before? This paper departs from the conjecture that an entrant may find it preferable to enter and compete instead of merging with the incumbent and matching product characteristics if her product is sufficiently strongly differentiated from the incumbent's. The conjecture rests on the observation that, after a merger, the jointly offered products generally become more unified. The conjecture is shown to hold in a simple linear city model of product differentiation under certain conditions. A variant of the conjecture is also shown to hold in a special case of two-dimensional characteristics with continuous demand. The intuition for the results is that entry without merging product characteristics allows to extract more consumer rent when consumers are sufficiently sensitive to product differentiation. The simple models exhibits limitations, however. I therefore indicate possible directions for future research.

The past decade has been marked by a new surge in merger activity. Among the 30 largest mergers ever, 23 took place after 1995 (Wall Street Journal 1998). Many mergers have been mergers across national borders.

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The advantages of integrating two firms' operations may be numerous, ranging from raising production or sales efficiency, to reducing competition, to sharing risk. Simultaneously, the number of alliances in industry has gone up significantly (Harbison and Pekar 1998). As opposed to mergers, alliances are legally much less clear cut, they may be transient or limited to small parts of the firms' operations. Even though they go mostly unnoticed, their economic impact is not negligible. Why so much matching and mating now, and less before? Given the fact that tariffs, transport costs and barriers to entry for international entrants have been reduced significantly around the globe, this looks even more startling. Conditions for entry now seem more inviting than ever. Why then do we observe more mergers now and less entry?

One hypothesis is that product space is more crowded now than it was some decades ago. When international firms, who had been serving national markets almost exclusively, started to expand globally, product characteristics were very different across national markets. After entry, the newly introduced foreign products met preferences of some consumers more closely than did the incumbents' varieties before. Under this hypothesis, entry would occur in 'young markets' where product space is still vastly uncovered, and mergers would occur in 'older markets' with crowded product space.

Another possible explanation is that many products are inseparable from their 'delivery' or require after-sales services. If that is the case, entrants need to either build-up a sales and services network of their own or to merge with the incumbent in order to access its network. Three examples underpin this idea. The product of a bank or any other financial intermediary is financial services. These are often inseparable from their delivery through local branches or direct banking networks. A bank merger will allow the merging banks to offer the entire product range after merger through one network. Similarly, manufacturers of investment goods, machinery, or personal computers may want to join their sales and after-sales networks but to retain their product range unchanged afterwards. Finally, there is also a group of examples without any geographic interpretation. Manufacturers of computer chips tend to join the research and development for the next generation chip but to implement the chips in products of different use. Car manufacturers tend to join the production of some of the parts of their cars but to keep other components separate.

From the point of view of production efficiency, all these examples could be explained readily: If R&D or production of key components involve fixed costs, if the setup and advertising of sales and after-sales networks involve fixed costs, mergers can exploit the increasing returns to scale. However, one important aspect of the merger is overlooked in this explanation. Some elements of the product characteristics are merged, too. Cars of merged companies may differ in the chassis, but all come with the same engine. The financial services of a bank merger may remain strongly differentiated, whereas the service components or credit requirements may become the same. These observations rest on the fact that product characteristics are multi-dimensional, they may become unified in some respects after the merger or alliance, but remain differentiated in other respects.

A branch of economic literature on location in product (or geographic) space has started to explore multi-dimensional aspects of product characteristics more thoroughly (most prominently Irmen and Thisse 1998; but also Vandenbosch and Weinberg 1995, Tabuchi 1994, Swann 1990). The merger versus entry literature has mostly neglected the fact that a merger often results in a reduction of the available product characteristics. Among the key reasons for mergers or acquisitions the following have been stressed: Mergers to monopolize homogeneous product industries or associations to decrease production and raise price (Gowrisankaran 1999, Kamien and Zang 1990, d'Aspremont and Gabszewicz 1986); the reduction of R&D costs or compatibility costs (Bloch 1995); and mergers to exploit complementary assets (Teece 1980). Looking at mergers from the perspective of product characteristics may add yet another reasoning to the above list. As the analysis below will show, this perspective reveals consequences of mergers that may make them less attractive to firms than entry in certain circumstances.

Keeping products differentiated may allow firms to get close to many consumers' preferences and to extract more consumer rent when competition is imperfect. However, keeping products differentiated is also likely to result in less synergies and cost savings as compared to merging. As far as the decision to *enter* or to *merge* is concerned, a trade-off is likely to arise: When products are strongly differentiated and no merger occurs, consumers' preferences will be closely met which tends to increase profits due to more consumer rent extraction. In addition, price competition between the entrant and the incumbent will be the weaker the further apart their commodities are in product space. Thus, one choice is to *enter* (without merging) and to *extract* (consumer rent). On the other hand, when products are close, price competition between entrant and incumbent will be strong, consumers' preferences cannot be closely met, and profits are likely to be higher after a merger or alliance than under entry. Under a merger, product characteristics have to be matched so that some consumers may no longer buy, but the two firms avoid the profit reducing effects of price competition. Thus, the alternative choice is to *merge* (with the incumbent) and to *match* (the two products in selected aspects of their design, or to sell only one).

This paper states the described trade-off as a simple conjecture: *Given their product characteristics, firms will choose to enter and compete whenever their products are strongly differentiated, and to merge whenever their products are close in characteristics.* The idea is that product characteristics are expensive to develop and evolve under given consumer preferences and market size. When trade barriers fall, firms want to sell the products with their given characteristics in new markets. This paper proceeds in three steps. In the first section, I will investigate the conjecture in one-dimensional product space using Hotelling's linear city model.[1] This standard model provides a baseline and clarifies the conditions under which the conjecture holds true. In the second section, I will outline a model of continuous demand in two-dimensional product space. The possibility of continuous demand for substitutes has been mostly disregarded in the literature on product differentiation. The model will shed light on how continuous demand can overlay some of the binary choice features of other models. A modified version of the conjecture holds true in this model. Because of the special ingredients, however, the model should only be taken as a point of departure to two dimensions. The third section compares the two models, considers open questions in the literature, and briefly points out possible paths for future research.

# 1 First Results in One-Dimensional Product Space

Mainly four modeling approaches to *horizontal* product differentiation have prevailed in the literature. Three of them make the consumer's decision binary: "Buy if product characteristics are close enough to your preferences and prices are low enough. Don't buy otherwise." Consumers are typically assumed to be living along a continuous line or a circle. This makes demand continuous within ranges, and discontinuous at points. The three types of

[1] The derivations in this section and subsequent sections were done with the help of Mathematica 4.0 for students. Similarly, all plots were done in Mathematica.

models are in particular: First, Hotelling's linear city model, the predecessor of all horizontal differentiation models (Hotelling 1929). Second, Salop's (1979) circular city model. And third, as Helpman labels it, a 'Chamberlinian' model in which each consumer has a most preferred product type on a circle of varieties (?). Finally, there is a fourth modeling approach which does not endogenize the choice of product differentiation but takes the degree of differentiation as given. As opposed to the first three approaches, consumers do not face a binary decision to either purchase one good or the other. Instead, they consume all substitutes in a certain ratio depending on their relative prices, as long as these prices do not rise too far relative to all other commodities. This approach is most familiar from Bertrand competition with non-homogeneous substitutes. It has been extensively used in models with asymmetric information (e.g. Raith 1996, Sakai and Yamato 1989). Even though there has been some effort to compare and reconcile these approaches, they still stand beside each other somewhat unconnected (see Anderson, de Palma, and Thisse 1989 for an attempted integration of the first three modeling approaches).

## 1.1 Entry vs. Merger

Many of these models' applications consider differentiation along one dimension of product characteristics only. In this section, let us restrict attention to one dimension, too, and use Hotelling's linear city model to obtain some first results. The following section will add a second dimension. In the linear city, a continuum of consumers is living along a line of length one, and the consumers are uniformly distributed over this line. Consumer $k$ lives at position $x_k \in [0, 1]$ and has utility

$$U_k = \bar{s} - p_i - (l_i - x_k)^2 \cdot t, \tag{1}$$

where $\bar{s}$ denotes utility from consuming one unit of the good. In this model, all consumers have unit demands. If at all, they only purchase one variety of the good. There are two components of disutility: First, the consumer has to pay a price $p_i$ to firm $i$ from which she purchases. Second, firm $i$ is located somewhere along the unit line at $l_i \in [0, 1]$ and consumer $k$ has to travel over a distance $\|l_i - x_k\|$ to make the purchase. The disutility of traveling (the transportation cost) is $(l_i - x_k)^2 \cdot t$. Instead of this geographic interpretation, we could also choose to think of variety $i$ as being more or less close to the consumer's ideal variety $x_k$.[2]

For simplicity, there are only two firms. An incumbent in the domestic market, and an entrant from abroad. If no entry occurs, each consumer can only buy from the incumbent or choose to not make any purchase. If entry occurs, each consumer can choose between two firms, and to buy or not. Let's first consider the case of entry, and later the case of merger. Suppose the incumbent is located at $l_I = \lambda_I$ and let the entrant have a product characteristic $l_E = 1 - \lambda_E$. Then, $\lambda_E$ is the entrant's distance from point 1 on the unit line, and $\lambda_I$ is the incumbent's distance from point 0. Without loss of generality, assume that $1 - \lambda_E > \lambda_I$. After entry, consumer $k$ will purchase from the incumbent iff $\|\lambda_I - x_k\| \leq \|1 - \lambda_E - x_k\|$. Thus, all consumers to the left of some critical consumer at $\hat{x}$ will purchase from the incumbent, and all consumers to the right will buy the entrant's variety. The critical consumer's position is implicitly given by the indifference condition

$$p_I + (\hat{x} - \lambda_I)^2 t = p_E + (1 - \lambda_E - \hat{x})^2 t.$$

So,

$$\hat{x} = \frac{\lambda_I + (1 - \lambda_E)}{2} + \frac{p_E - p_I}{2t\left[(1 - \lambda_E) - \lambda_I\right]}. \tag{2}$$

The incumbent and the entrant will compete in prices (Bertrand competition). Both face the same constant marginal cost of production, $c$. Given a conjectured price choice $p_E$ of the entrant, the incumbent will choose

$$p_I^* \equiv \arg\max_{p_I} (p_I - c)\hat{x}.$$

If the incumbent raised price too high, some demand to the left of $\lambda_I$ would break away because some consumers would choose not to buy at all. Let's disregard this case, however. Since Bertrand competition is going to be fierce, prices will be considerably lower after entry than in the case of merger. We want to compare entry with merger. It is therefore safe to assume that $\bar{s}$ is sufficiently high so that all consumers to the left of $\hat{x}$ will buy in a Bertrand equilibrium after entry. (Whether all consumers will buy from a monopoly is a different matter and considered below.) Then

$$p_I^* = \frac{1}{2}\left[p_E + c + t\left((1 - \lambda_E)^2 - \lambda_I^2\right)\right].$$

[2] I choose quadratic disutility to avoid technical problems when I make choice of location endogenous later. The analysis in this section is exactly identical under linear transportation cost.

Similarly, given a conjectured price choice $p_I$ of the incumbent and assuming that $\bar{s}$ is high enough, the entrant will set

$$p_E^* \equiv \arg\max_{p_E}(p_E - c)(1 - \hat{x}) = \frac{1}{2}\left[p_I + c + t\left((1-\lambda_I)^2 - \lambda_E^2\right)\right].$$

Solving out for the mutually consistent prices $p_I^*$ and $p_E^*$, the unique Nash equilibrium of the Bertrand game becomes

$$p_I^{eql} = c + \frac{1}{3}t\left[3 + \lambda_I - \lambda_E\right]\left[(1-\lambda_E) - \lambda_I\right] \tag{3}$$

and

$$p_E^{eql} = c + \frac{1}{3}t\left[3 + \lambda_E - \lambda_I\right]\left[(1-\lambda_E) - \lambda_I\right]. \tag{4}$$

The according equilibrium profits for the incumbent and the entrant are

$$\Pi_I = \frac{1}{18}t\left[3 + \lambda_I - \lambda_E\right]^2\left[(1-\lambda_E) - \lambda_I\right] \tag{5}$$

and

$$\Pi_E = \frac{1}{18}t\left[3 + \lambda_E - \lambda_I\right]^2\left[(1-\lambda_E) - \lambda_I\right]. \tag{6}$$

Adding (5) and (6), the joint post-entry profits of the two firms become

$$\Pi_I + \Pi_E - F_E = t\left[1 + \left(\frac{\lambda_I - \lambda_E}{3}\right)^2\right]\left[(1-\lambda_E) - \lambda_I\right] - F_E.$$

When deriving these results, all consumers were assumed to buy in equilibrium. A necessary and sufficient condition for this is that $\bar{s} - \lambda_E^2 t - p_E^{eql} \geq 0$ for the 'first' consumer at point zero, and that $\bar{s} - \lambda_I^2 t - p_I^{eql} \geq 0$ for the 'last' consumer at point one. Equivalently,

$$\frac{\bar{s} - c}{t} \geq \begin{cases} \frac{1}{3}\left(3 + \lambda_I - \lambda_E\right)\left((1-\lambda_E) - \lambda_I\right) + \lambda_I^2 & \text{if } \lambda_I \geq \lambda_E \\ \frac{1}{3}\left(3 + \lambda_E - \lambda_I\right)\left((1-\lambda_E) - \lambda_I\right) + \lambda_E^2 & \text{if } \lambda_I < \lambda_E \end{cases}. \tag{7}$$

If there were some fixed costs of entry, $F_E$, the entrant would choose to enter as long as $\Pi_E > F_E$. What about the incentive to merge with the incumbent? As long as $\Pi_E > F_E$, the entrant can credibly threaten to enter. Then, a merger will be more lucrative for both firms iff

$$\Pi_E + \Pi_I - F_E \leq \Pi_M - F_M, \tag{8}$$

where $\Pi_M$ and $F_M$ are profits and fixed costs of a merger, respectively. If condition (8) fails, then it is *impossible* to re-shuffle merger profits among the two firms in a way that both will prefer a merger over competition. In this case, entry and competition will occur, as long as $\Pi_E > F_E$. If condition (8) holds, however, entry will not occur; firms will merge and match their product characteristics.

## 1.2 A merger's price choice and profits

A monopoly's or a merger's price choice in the linear city model is complicated by the fact that demand drops discontinuously when price surpasses a certain threshold level because then some consumers choose not to buy at all. Table 1 shows the results.

The optimal price choice crucially depends on the ratio between potential consumer surplus $\bar{s}-c$ and transportation cost $t$. In ranges where the surplus to transportation cost ratio $\frac{\bar{s}-c}{t}$ is low, consumers at the city limits stop buying the good even for relatively low price premia over marginal cost. In this range, a monopoly makes most profits when raising price so far that only a fraction of consumers buys. When the ratio $\frac{\bar{s}-c}{t}$ is high, however, the monopoly is careful to keep price exactly at the level where the entire market is just covered. A derivation of the results in table 1 is given in the appendix (p. 27). In several ranges, seemingly more than one optimal price choice exists (there are two or four roots to the problem). The table reports the solutions which make the price choice continuous, so that prices and profits rise steadily over the entire range of $\frac{\bar{s}-c}{t} \in [0, \infty)$.

All results in table 1 are given with respect to some location $\lambda_M$ of the merger in product space. As can be shown by differentiating optimal profit with respect to the merger's location, merger profits are maximal when locating at the center of the city, $\lambda_M = \frac{1}{2}$. (In the lowest range of $\frac{\bar{s}-c}{t}$, the monopoly is indifferent because in this range demand breaks away on both sides of the market anyway.) In order to maximize profits, the merger will choose between the two possible locations, $\lambda_E$ and $\lambda_I$, and pick the one that is closer to one half. Recall that we assumed, without loss of generality, that the entrant is located to the right of the incumbent, $1 - \lambda_E \geq \lambda_I$. Therefore

Table 1: **Price Choice and Optimal Profits of a Merger**

| Range | $\frac{\bar{s}-c}{t} \in$ | $p_M =$ | $\Pi_M =$ | $\Pi_M \in$ |
|---|---|---|---|---|
| 3 | $[0, 3\lambda_M^2]$ | $\frac{2\bar{s}+c}{3}$ | $4t\left(\frac{\bar{s}-c}{3t}\right)^{\frac{3}{2}}$ | $[0, 4t\lambda_M^3]$ |
| 2c | $(3\lambda_M^2, 5\lambda_M^2]$ | $\bar{s} - \lambda_M^2 t$ | $2t\lambda_M\left[\frac{\bar{s}-c}{t} - \lambda_M^2\right]$ | $(4t\lambda_M^3, 8t\lambda_M^3]$ |
| 2b | $(5\lambda_M^2, (1-\lambda_M)(3-\lambda_M)]$ | $\frac{2\bar{s}+c}{3} - \frac{2}{3}t\lambda_M\left(\frac{\lambda_M}{3} - \sqrt{3\frac{\bar{s}-c}{t} + \lambda_M^2}\right)$ | $X^{\dagger)}$ | $(8t\lambda_M^3, 2t(1-\lambda_M)]$ |
| 2a, 1 | $((1-\lambda_M)(3-\lambda_M), \infty]$ | $\bar{s} - (1-\lambda_M)^2 t$ | $t\left[\frac{\bar{s}-c}{t} - (1-\lambda_M)^2\right]$ | $(2t(1-\lambda_M), \infty)$ |

†) where $X \equiv \frac{2}{27}t\left(3\frac{\bar{s}-c}{t} - \lambda_M^2 + \lambda_M\sqrt{3\frac{\bar{s}-c}{t} + \lambda_M^2}\right)\left(3\lambda_M + \sqrt{3\frac{\bar{s}-c}{t} + 2\lambda_M^2 - 2\lambda_M\sqrt{3\frac{\bar{s}-c}{t} + \lambda_M^2}}\right)$

the optimal location choice for the merger can simply be written as $\lambda_M = \max[\lambda_I, \lambda_E]$. This establishes the value of profits of the merged firm.

### 1.3 When entry is more profitable than merger

Knowing both maximal profits of a merger and the sum of equilibrium profits that incumbent and entrant can make after entry, we can investigate under which conditions entry and competition will be more profitable. As it will turn out, entry occurs in equilibrium only if the ratio $\frac{\bar{s}-c}{t}$ is low. For higher levels, a merger can raise price strongly, and still extract so much consumer surplus that entry is inferior irrespective of how far apart the products are in product space. Price competition would always be too harmful to profits. However, in ranges 3 and 2c of the surplus to transportation cost ratio (table 1) entry is more lucrative if products are far enough apart (and entry cost $F_E$ do not exceed merger cost $F_M$ by too much). In these two ranges, the condition for a merger to be preferable for both firms (8) fails whenever products are sufficiently distant. Proposition 1 states this in a more formal manner.

**Proposition 1** *Suppose product characteristics are given, incumbent and entrant compete in price after entry, and the entry threat is credible ($\Pi_E > F_E$). Then*

*i) For $F_E = F_M$:*
*If $\frac{\bar{s}-c}{t}$ falls in ranges 3 or 2c in table 1 (p. 9), then there is a set of distances between $\lambda_E$ and $\lambda_I$, in which at least one firm prefers entry over merger. Formally, when $\lambda_E = \lambda_I \to \frac{1}{2}$, the necessary merger condition (8) fails.*
*On the other hand, if the ratio $\frac{\bar{s}-c}{t}$ approaches the upper bound of range 2b, entry can never be more profitable than merger.*

*ii) As the difference $F_E - F_M > 0$ increases,*
*merger becomes more profitable and can now be preferable in $\frac{\bar{s}-c}{t}$ ranges 3 and 2c, where entry was dominant before.*

*iii) Similarly, as the difference $F_E - F_M < 0$ falls,*
*entry becomes more profitable and can now be preferable to merger even in $\frac{\bar{s}-c}{t}$ ranges 2b and 2a, where merger was dominant before.*

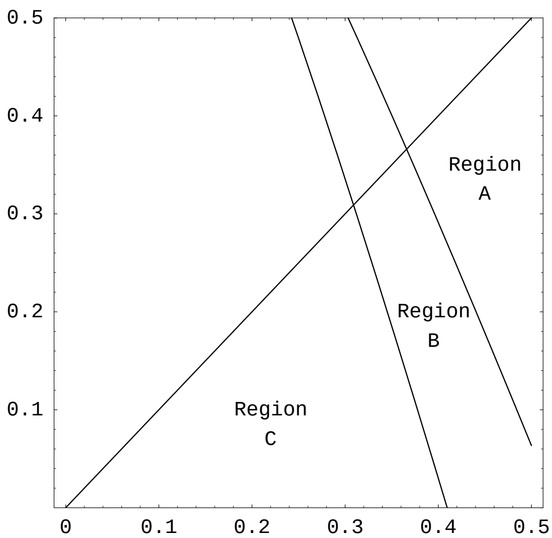


Figure 1: **Compatible Regions of $\lambda_I$ and $\lambda_E$**

The statement of the proposition can be strengthened. It is, in principle, possible to derive the precise geometric boundaries of the set of $\lambda_E$-$\lambda_I$-combinations for which entry occurs. Time did not permit to do so. Similarly, the exact boundary value of the ratio $\frac{\bar{s}-c}{t}$ beyond which a merger will always be more profitable can also be calculated in principle. How such results can be obtained will become more clear in the following proof of proposition 1.

**Proof.** To derive proposition 1, let's start with $F_E = F_M$. In addition, let's concentrate on the case where the incumbent is closer to the city center (point one half) than the entrant. That is, let's assume that $\lambda_I \geq \lambda_E$ and $\lambda_I, \lambda_E \in (0, \frac{1}{2})$.This simplifies the analysis, but the following results will hold for the remaining cases, too. In figure 1, the only permissible regions under this assumption lie below the 45° line. Also, under the assumption $\lambda_I \geq \lambda_E$, the product choice of the merger will be $\lambda_M = \lambda_I$. For each range of $\frac{\bar{s}-c}{t}$ in table 1, we now have to ask: Is it possible that condition (8) holds, that is, is it possible that the joint post-entry profits of the two firms exceed merger profits? And if so, under what conditions? Since we assumed that all consumers buy in a post-entry equilibrium, condition (7) mandates that the relation $\frac{1}{3}(3 + \lambda_I - \lambda_E)((1 - \lambda_E) - \lambda_I) + \lambda_I^2 \leq \frac{\bar{s}-c}{t}$ must hold within each range of $\frac{\bar{s}-c}{t}$ (for $\lambda_I \geq \lambda_E$). It can be readily verified that this condition will be violated beyond the upper bound of range 2b: It namely must be the case that $\frac{1}{3}(3+\lambda_I-\lambda_E)((1-\lambda_E)-\lambda_I)+\lambda_I^2 < (1-\lambda_I)(3-\lambda_I)$ for $\lambda_E, \lambda_I > 0$. Hence, merger will always be preferable in range 2a (and 1). It will also be strictly preferable beyond some interior point in range 2b. This proves the last statement in part i) of the proposition.

In the lower ranges of $\frac{\bar{s}-c}{t}$, however, entry can be the dominant choice. Take range 3 first. Since $\frac{\bar{s}-c}{t}$ must fall within the boundaries of range 3 and satisfy condition (7), the surplus to transportation cost ratio must lie in the reduced range $\frac{\bar{s}-c}{t} \in [\frac{1}{3}(3+\lambda_I-\lambda_E)((1-\lambda_E)-\lambda_I)+\lambda_I^2, 3\lambda_I^2]$. For being in that range, we must have $\frac{1}{3}(3+\lambda_I-\lambda_E)((1-\lambda_E)-\lambda_I)+\lambda_I^2 \leq 3\lambda_I^2$, which is satisfied iff $\lambda_E \geq (2-\sqrt{1+2\lambda_I+7\lambda_I^2})$. This condition is depicted as region A in figure 1. When is entry the dominant choice in this region? Note that the distance between the two commodities in product space, $\|(1-\lambda_E)-\lambda_I\|$, is maximized the closer $\lambda_I$ and $\lambda_E$ get to the origin. In fact, if we drew 'iso-differentiation' contours in $\lambda_I$-$\lambda_E$-space, these contours would all be parallel lines, perpendicular to the 45° line. If we restrict our attention to locations where $\lambda_E = \lambda_I$, then a move along the 45° line in figure 1 to the origin corresponds to an increase in differentiation. It is therefore natural to check whether joint profits after entry exceed merger profits at the point of intersection between the lower bound of region A and the 45° line. At this point,

$$\lambda_E = \lambda_I = \frac{\sqrt{3}-1}{2}.$$

If $\frac{\bar{s}-c}{t}$ is at its maximal level within this range ($\frac{\bar{s}-c}{t} = 3\lambda_I^2$), the corresponding merger profits attain their highest level within the range: $\Pi_M^{\max} = 4t\lambda_I^3$. Thus the minimal difference between joint post-entry and merger profits becomes

$$\begin{aligned}\Pi_E + \Pi_I - \Pi_M^{\max} &= t\left[1+\left(\frac{\lambda_I-\lambda_E}{3}\right)^2\right][(1-\lambda_E)-\lambda_I] - 4t\lambda_I^3 \\ &= t(7-4\sqrt{3}) > 0.\end{aligned}$$

There must hence be an area of strictly higher joint post-entry profits around that point in region A. The boundaries of that region remain to be derived; they will depend on the level of $\frac{\bar{s}-c}{t}$ (which makes the derivation hard). In figure 2, a zero-contour of the function $\Pi_E + \Pi_I - \Pi_M^{\max}$ in (9) is depicted as the curve that divides region A. In the area to the South-West of that curve entry is more profitable, in the area to the North-East a merger is preferable. Note that $\Pi_M^{\max} = 4t\lambda_I^3$ is only achieved when $\frac{\bar{s}-c}{t}$ reaches the upper bound of range 3, $\frac{\bar{s}-c}{t} = 3\lambda_I^2$. For lower values of $\frac{\bar{s}-c}{t}$, the zero-contour curve would lie even further to the North-East. The plot therefore suggests that a large part of region A makes entry more desirable than merger. It also suggests that, within region A, the degree of product differentiation needs to be large enough ($\lambda_E$-$\lambda_I$-combinations in the South-West) to make entry more desirable than merger. An algebraic proof remains to be given.

Similar arguments apply to range 2c. For this range and condition (7) to apply, we must have $\frac{\bar{s}-c}{t} \in [\frac{1}{3}(3+\lambda_I-\lambda_E)((1-\lambda_E)-\lambda_I)+\lambda_I^2, 5\lambda_I^2]$. Hence, $\frac{1}{3}(3+\lambda_I-\lambda_E)((1-\lambda_E)-\lambda_I)+\lambda_I^2 \leq 5\lambda_I^2$, which is satisfied iff $\lambda_E \geq \frac{1}{7}(2-\sqrt{1+2\lambda_I+13\lambda_I^2})$. The corresponding $\lambda_E$-$\lambda_I$-combinations are depicted as region B in figure 1. Evaluating the difference between joint post-entry profits and merger profits at the maximum distance in region B, we obtain

$$\lambda_E = \lambda_I = \frac{\sqrt{5}-1}{4}$$

and

$$\begin{aligned}\Pi_E + \Pi_I - \Pi_M^{\max} &= t\left[1+\left(\frac{\lambda_I-\lambda_E}{3}\right)^2\right][(1-\lambda_E)-\lambda_I] - 8t\lambda_I^3 \\ &= \frac{t}{2}(7-3\sqrt{5}) > 0.\end{aligned}$$

Again, there must be an area around this point where entry is more profitable than merger. As the curve that divides region B in figure 2 suggests, the area of preferable entry (which lies to the South-West of the curve) seems to be quite large even for $\frac{\bar{s}-c}{t} = 5\lambda_I^2$.

Finally, these results change if $F_E \neq F_M$. For a large positive difference $F_E - F_M > 0$, merger becomes more desirable even in ranges 3 and 2c. For a large negative difference $F_E - F_M < 0$, entry becomes more desirable even in ranges 2b and 2a. ■

My initial conjecture that, given their product characteristics, firms will choose to enter and compete whenever their products are strongly differentiated, is thus confirmed in the one-dimensional case under certain circumstances. The conjecture is proven to be right in markets where consumers are highly sensitive to product characteristics (where $\frac{\bar{s}-c}{t}$ is low). The conjecture is refuted for markets with consumers who are relatively insensitive

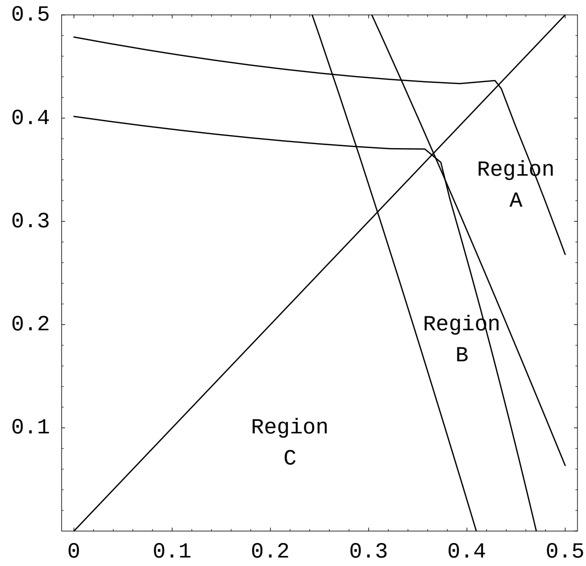


Figure 2: **Regions of $\lambda_I$ and $\lambda_E$ for which Entry Occurs**

to product characteristics (where $\frac{\bar{s}-c}{t}$ is high). A general problem with the Hotelling model of product differentiation is that it does not lend itself easily to empirical testing. The model cannot be calibrated. Empirically testable implications remain to be worked out.

The assumption that product characteristics are given to firms may seem restrictive at first sight. In practice, however, this need not be a restrictive assumption. In both heterogeneous and homogeneous goods markets, firms struggle to establish brands. This can be very costly. Even when a product is updated, or a new generation of a product is launched, firms choose to retain certain characteristics from older versions so that their brand will remain recognizable. This happens in car manufacturing as much as in software programming. Key product characteristics are only changed slowly, and it seems to be an admissible approximation to consider product characteristics as given for some time horizon. As is well known, the two firms would choose to locate at the extremes of the unit line if they were entirely free to reconsider their product design—in order to reduce price competition to its minimal level. That is, the two firms would choose maximum differentiation. The incentives for entry remain very similar when firms *can* choose their location. Whenever entry occurred under given characteristics, firms will now choose strong differentiation and also decide not to merge. However, the result of maximum differentiation only holds in one dimension. If product characteristics were multi-dimensional, Hotelling's original claim that both firms will locate in the *center* in order to capture possibly much consumer demand, can be proven to be correct in all dimensions but one (Irmen and Thisse 1998). Hence, considering location choice in one dimension can be misleading. The more a multi-dimensional approach is needed.

If there is more than one entrant, the above proposition has to be refined. As long as $\Pi_E > F_E$, entrants can credibly threaten to enter, and will therefore be able to blackmail their way into a merger. This process will not stop until the last entrant has joined, provided that all entrants have *sunk* their cost of designing a variety before. All firms around the world that can offer a variety of the respective good will thus be able to join a merger. However, firms that did not yet sink the cost of designing a variety will at some point choose to no longer design a variety, when profits per member of the merger have fallen sufficiently.

An interesting extension of the above model might be to consider growing markets. Suppose demand grows, and certain varieties become more demanded than others due to some complementarities. For example, certain types of cars may become more suitable for changing commuter needs, different types of financial services may become more adequate under the deepening of financial markets. This type of growth could be treated like an extension of the variety line to the right (beyond one) when varieties of higher order on the unit line become more attractive under growth. To make modeling easier, this growth pattern could be translated into a simultaneous shift of the firms' locations to the left and an increase in $\bar{s}$. If firms incur some cost of relocation, they would redesign their variety at discrete points of time. The relocation costs could be increasing in the degree of change, given that new brands need to be established in the market. Under this kind of market growth, the decision problem to enter or to merge changes. I conjecture that, given new demand is bound to 'arrive' continuously, retaining two differentiated goods might become more lucrative, and entry more likely. Firms could even choose to locate beyond point one and to wait for future demand if costs of redesign are very high. Such a result would support the hypothesis stated in the introduction that entry is more likely in 'young' (and fast growing) markets—but with a different motivation.

# 2 Some Observations in Two Dimensions under Continuous Demand

The conjecture that entry is preferable when products are strongly differentiated remains to be analyzed in multi-dimensional product space. The initial conjecture is essentially a statement about multiple product characteristics; products may become unified in some respects after the merger or alliance, but remain differentiated in other respects. As psychological studies have shown early on (Miller 1956), our ability to make an informed choice between alternatives with more than around seven characteristics is extremely limited. Yet, it seems implausible, too, to restrict our attention to only one characteristic. When we buy a car, we look at its safety, its fuel consumption, and its projected maintenance costs—if not more. Similarly we pay attention to several characteristics in our choice of a computer, or our bank. Finally, not all these decisions are binary in nature. It is not the case that we only buy one type of the same good. We can own more than one computer, one at home, one at work, and a laptop. We can drive more than one car. We may choose to have accounts at several banks, we usually buy insurance from more than one insurer. For non-durable goods, this is even more obvious: We tend to eat more than one type of cereals for breakfast. So far, the product differentiation literature (the first three approaches mentioned at the beginning of section 1, p. 4) and the literature on competition with heterogeneous, but not perfectly substitutable products (the fourth approach) remain unrelated. This section sets out to explore a model with product differentiation à la Hotelling and with a given degree of differentiation in a second dimension. Since I treat product design as exogenous, the assumption of given differentiation in a second dimension is not problematic. There is no clear way, however, in which it could be endogenized in this type of model.

## 2.1 Utility, demand and a given degree of differentiation

Suppose consumers care about three goods: two of them are the substitutes that we are concerned with, such as desktop computers and laptops, and a third good represents all the rest. Each consumer $k$ could have a utility function of the form

$$U_k(q_0, q_I, q_E) = q_0 + \alpha(q_I + q_E) - \frac{1}{2}\beta(q_I^2 + 2\theta q_I q_E + q_E^2). \tag{9}$$

Now $q_i$ denotes the consumed quantity of good $i \in \{0, I, E\}$, and $\alpha$, $\beta$, and $\theta$ are scalar parameters. For reasons that will become clear later, $\theta$ is restricted to $\theta \in (-\beta, \beta)$ if $\|\beta\| \leq 1$ and to $\theta \in (-1, 1)$ if $\|\beta\| > 1$. This utility specification will lend itself to linear demand functions with goods $E$ and $I$ being substitutes for $\theta > 0$ and complements for $\theta < 0$. Let's now assume that there is a second dimension of product differentiation. Suppose in particular that each of the two substitutes comes more or less close to each consumer $k$'s most preferred variety. In our geographical interpretation, each of the two goods can be sold from a different location. Then, a natural extension of the budget constraint is to include transportation cost:

$$q_0 + \left[p_I + (l_I - x_{k,I})^2 t\right] q_I + \left[p_E + (l_E - x_{k,E})^2 t\right] q_E \leq \bar{Y}_k. \tag{10}$$

Here $p_i$ denotes the price of good $i \in \{I, E\}$, and $p_0 = 1$ makes the 'background good' the *numéraire*. As before $l_i$ is the location of firm $i$ in product space, $l_i \in [0, 1]$. Now $x_{k,i}$ is consumer $k$'s most preferred variety of good $i$ and, in principle, the favorite variety can differ for both products: $x_{k,I} \neq x_{k,E}$. In this specification of the budget constraint, transportation cost increase in quantity (which makes the optimization problem smooth). A consumer suffers from the distance to its most preferred variety, each time she consumes the product. Or a consumer has to incur the transportation cost for each quantity that she purchases. In order to keep the analysis simple, I will restrict attention to the case where a consumer's most preferred variety is located at the same point on the unit line for both goods: $x_{k,I} = x_{k,E} = x_k$. Theoretically, there is no reason why this has to be so in abstract product space. If we adopted a more geographical interpretation, however, this restriction is natural. A consumer only lives at one place at a time.

The consumer maximizes utility (9) with respect to the budget constraint (10) for $q_I$, $q_E$, and $q_0$. In optimum, consumer $k$ chooses

$$q_I^*(x_k) = \frac{1}{\beta(1-\theta^2)}\left[\alpha(1-\theta) - p_I + \theta p_E - (l_I - x_k)^2 t + \theta(l_E - x_k)^2 t\right], \tag{11}$$

$$q_2^*(x_k) = \frac{1}{\beta(1-\theta^2)}\left[\alpha(1-\theta) - p_E + \theta p_I - (l_E - x_k)^2 t + \theta(l_I - x_k)^2 t\right], \tag{12}$$

and

$$q_0^*(x_k, \bar{Y}_k) = \bar{Y}_k - \left[p_I + (l_I - x_{k,I})^2 t\right] q_I^*(x_k) + \left[p_E + (l_E - x_{k,E})^2 t\right] q_2^*(x_k), \tag{13}$$

if $\bar{Y}_k$ is large enough so that an interior solution is feasible. For our quasi-linear preferences, an interior solution is guaranteed iff $q_0^*(x_k, \bar{Y}_k) \geq 0$. The second order conditions to this problem mandate that $\theta \in (-\beta, \beta)$.

If we let consumers be uniformly distributed over the unit line again, each firm $i$ faces a demand of

$$\begin{aligned} D_i &= \int_0^1 q_i^*(x)dx \\ &= \frac{\frac{1}{3}(3\alpha - t)(1-\theta) - p_i + \theta p_{-i} + t\left[l_i(1-l_i) - \theta l_{-i}(1-l_{-i})\right]}{\beta(1-\theta^2)}. \end{aligned}$$

Here, $p_i$ and $l_i$ denote firm $i$'s price and location, whereas $p_{-i}$ and $l_{-i}$ denote the other firm's price and location, respectively. For demand to be positive, we require $\theta \in (-1, 1)$. Note that the location terms $l_i(1-l_i)$ and $l_{-i}(1-l_{-i})$ are maximal when $l_i$ and $l_{-i}$ are at one half. They minimal at zero and one. So, as in the basic Hotelling model, a firm captures more demand for its own product if it is located centrally. The location term of the other firm, however, enters negatively. The reason is that the goods are substitutes for $\theta > 0$, and a central location of the other firm increases demand for the other good which reduces demand for the firm's own product. Given the substitutability of the two goods, each firm would now like the other firm to locate far away. In the basic Hotelling model, the desirability of maximal differentiation came from price competition (which is lowest under maximal differentiation along the unit line). Now, there is an additional force to differentiate strongly along the unit line if the goods are substitutes. (On the other hand, if the goods are complements, the drive to differentiate is weakened.) This will also affect the initial conjecture that entry is preferable whenever products are strongly differentiated. When the goods are substitutes, entry does less harm to the incumbent's profits the closer to the city limit the entrant is located.

For given product characteristics, firms compete in price. Each firm maximizes profits by choosing the optimal price $p_i$, given the conjectured price choice $p_{-i}$ of its competitor and its own cost $c_i$:

$$\begin{aligned} p_i^* = \arg\max_{p_i} \Pi_i = \arg\max (p_i - c_i) D_i = \frac{1}{6}\big(&3(\alpha + c_i) + 3\theta p_{-i} \\ &- t - \theta(3\alpha - t) + 3t\left[l_i(1-l_i) - \theta l_{-i}(1-l_{-i})\right]\big). \end{aligned} \tag{14}$$

The Nash equilibrium of this Bertrand game is

$$p_i^{eql} = \frac{1}{3(4-\theta^2)}\big((3\alpha - t)(2-\theta(1+\theta)) + 6c_i + 3\theta c_{-i} \\ + 3t\left[(2-\theta^2)l_i(1-l_i) - \theta l_{-i}(1-l_{-i}\right]\big), \quad (15)$$

with equilibrium profits

$$\Pi_i^* = \frac{1}{9\beta(1-\theta^2)(4-\theta^2)^2}\big\{(3\alpha - t)(2-\theta(1+\theta)) - 3c_i(2-\theta^2) + 3\theta c_{-i} \\ + 3t\left[(2-\theta^2)l_i(1-l_i) - \theta l_{-i}\,(1-l_{-i})\right]\big\}^2. \quad (16)$$

## 2.2 A monopoly's decision

As a monopoly, the merger would exploit the (monetary) externality that raising price has on demand for the other good. A merger maximizes the profit for both of its production lines $l_i$ simultaneously and chooses prices $p_i$, $i \in \{I, E\}$:

$$p_i^M = \arg\max_{p_i} \Pi_i = \arg\max (p_i - c_i)D_i = \frac{1}{6}\left(3(\alpha + c_i) - t + 3t\left[l_i(1-l_i)\right]\right). \quad (17)$$

Due to the differentiation in a second dimension (measured by $\theta$) there are two distinct products now. This possibility did not arise in the model of the previous section.

The merger has several options to integrate products after merging. First, it can shut down one product line altogether. This could be desirable if the substitutability of the products were very strong and even a monopoly would compete against its own products—an impossible case in our framework. Second, the monopoly could keep offering both products with their original characteristics in all dimensions. Third, the monopoly can choose to unify products along one dimension. This is most profitable when one product is far from the center in one respect and more consumer rent can be extracted by offering both products at the more central location. These alternatives still remain to be worked out in detail. For now I will restrict attention only to the third case because it is most comparable to the results of the model in section 1. So, the merger chooses one location for both products, $l_M$. Then,

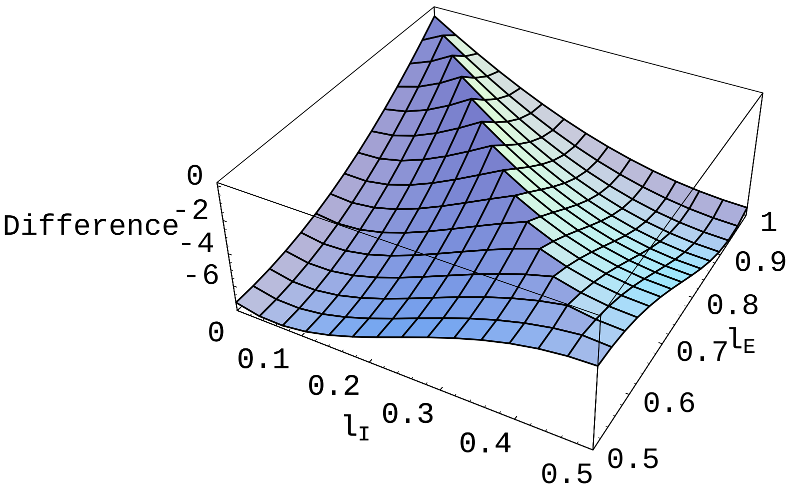


Figure 3: **Difference betw. Post-Entry and Merger Profits,** $\theta = \frac{1}{2}$

profits in optimum are:

$$\Pi_M = \frac{1}{36\beta(1-\theta^2)} \big\{ 9(c_E - c_I)^2 - 2(1-\theta)\,(3\alpha - t\,[1 - 3l_M(1-l_M)]) \\ \times (3(c_I + c_E - \alpha) + t\,[1 - 3l_M(1-l_M)])\,\big\}. \quad (18)$$

Merger profits (18) are maximized when $l_M = \frac{1}{2}$, unless transportation costs become too high.[3] When merging, the firms therefore compare their two products with respect to their characteristic $l_i$. The characteristic that is closer to one half gets selected for both products. Or, in geographic space, the merger places its sales outlet for both products close to the city center.

[3]There are three roots that solve the first order condition for location choice (maximizing (18) w.r.t $l_M$): $l_M = \frac{1}{2}$, and $l_M = \frac{1}{2} \pm \sqrt{12\alpha - t - 6(c_I + c_E)}/\sqrt{12t}$. The latter two roots are local minima, whereas $l_M = \frac{1}{2}$ is a local maximum. It is the global maximum if $t \leq 3(2\alpha - (c_I + c_E))/(5 - 18(1-l_M)l_M)$. As $t$ increases beyond that, $l_M = \frac{1}{2}$ remains the global maximum for some range. But as $t$ increases even further, locating at one of the *extremes* becomes optimal. This result is curious and may deserve further investigation. It is also closely related to the later findings in this section.

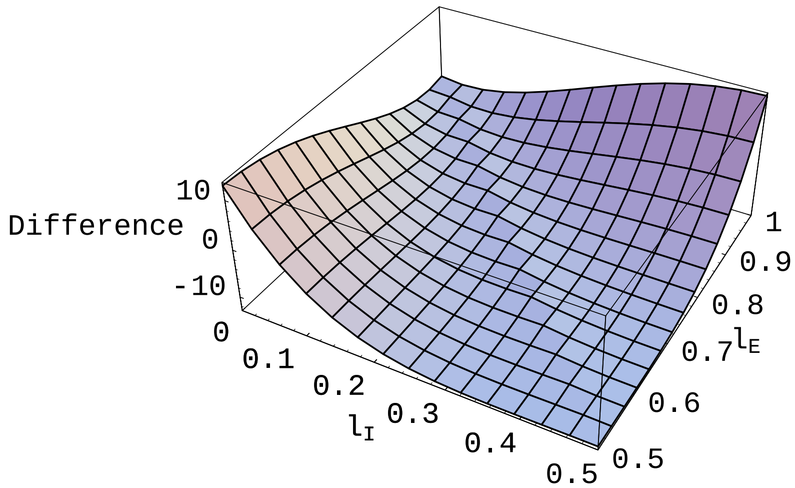


Figure 4: **Difference betw. Post-Entry and Merger Profits,** $\theta = \frac{4}{5}$

## 2.3 The incentives for merger and entry

Now we are in a position to compare profits after entry and profits of the merger. Let's neglect the existence of fixed costs of entry or merging to keep matters simple. Then, the difference between post-entry and merger profits $\Pi_E + \Pi_I - \Pi_M$ is a function of the position of the two firms in product space, $l_I$ and $l_E$, and the substitutability parameter $\theta$. The analysis turns out to be subtle. Several cases have to be distinguished. Therefore, I restrict attention here to the case where the incumbent is located to the left of the city center, and the entrant to the right ($l_I \in [0, \frac{1}{2})$ and $l_E \in (\frac{1}{2}, 1]$). This case is most comparable to the model in the previous section. In addition, I will only give graphical explanations. Algebraic proofs remain to be worked out.

Figures 3 and 4 plot $\Pi_E + \Pi_I - \Pi_M$ as a function of $l_I$ and $l_E$ for two different parameter values of $\theta$: $\theta = \frac{1}{2}$ and $\theta = \frac{4}{5}$.[4] The shape of the curves

[4] The exact functional form underlying the graphs in figures 3 and 4 is $\Pi_E(l_I, l_E) + \Pi_I(l_I, l_E) - \Pi_M(l_M)$, where $l_M \equiv \frac{1}{2} + \min[\|l_I - \frac{1}{2}\|, \|l_E - \frac{1}{2}\|] \times \text{sign}[\|l_E - \frac{1}{2}\| - \|l_I - \frac{1}{2}\|]$. This functional form picks the value $l_M = l_I$ whenever it is closer to one half and $l_M = l_E$ otherwise. It is not well defined at $l_E = l_I$ where the sign function jumps to zero. The

is most sensitive to different values of $\theta$. (A strong increase in $t$ has an effect similar to an increase in $\theta$.) Consider figure 3 first. The initial conjecture of this paper states that entry is more likely to occur when products are strongly differentiated. In figure 3, products are least differentiated at the Southern tip of the graph where $l_I = l_E = \frac{1}{2}$. They are most differentiated at the Northern tip of the graph where $l_I = 0$, $l_E = 1$. If we were to draw 'iso-differentiation' lines between these to extremes, they would all run parallel to the West-East diagonal of the graph. As the conjecture suggested, the difference between post-entry and merger profits increases indeed when products are more differentiated, that is, when we move from South to North in our graph. However, as can be read off the graph as well, the difference never exceeds zero. That is, for $\theta = \frac{1}{2}$ (and a broad range of values of the other parameters), merger is generally preferable. Playing around with the other parameter values and $\theta$ in a neighborhood of $\frac{1}{2}$ did not alter this heuristic finding. A general statement about the lack of entry in this model remains to be derived. One reason why merger could be generally preferable to entry in this model may be that demand is less responsive to an increase in price than in the Hotelling model. Whereas demand breaks away discontinuously at the city limits when a monopoly increases price too far in the Hotelling model, demand is reduced continuously in the quasi-linear utility model of this section. Now, consumers slowly increase demand for the third good in the background whenever price or transportation cost rise. This continuity leaves the merger with more demand when it unifies characteristics and starts charging monopoly prices than in the Hotelling world.

Things change dramatically when $\theta$ rises high. An increase in $\theta$ means that the two goods become closer and closer substitutes in the second dimension. For $\theta = 1$, they would be perfect substitutes as can be seen from (9). This case is not well defined in our framework. For high values of $\theta$ below one, such as $\theta = .9$ or $\theta = .95$, the environment for our two firms changes heavily. Now, entry can dominate merger. However, the circumstances under which entry becomes more preferable than merger fit the conjecture only partially. As can be seen from figure 4, entry is more preferable than merger whenever one firm is located close to the city limit. Surprisingly, however, *joint* post-entry profits are highest not when both firms locate at the city limits, but when one firm locates in the center and the other firm at the city limit. The

graphs, however, are drawn smoothly such that this oddity is neglected. The parameter values other than $\theta$ are: $c_I = c_E = 10$, $\alpha = 20$, $\beta = 1$, $t = 20$.

reason has to do with the negative (monetary) externality of a close substitute on demand for the own good. As mentioned before, this effect is new in the framework of this section and cannot arise in the simple Hotelling model. When goods are substitutes in the second dimension ($\theta > 0$), a central location of the competitor is not only bad due to increased price competition but also because of a demand effect. When a firm is centrally located, demand for its good is high, which reduces demand for the other product. Each firm would like to locate in the center itself, but would like the other firm to locate far away in order to capture possibly much demand for its own good. This phenomenon is reflected in the sum of post-entry profits. When $\theta$ gets closer to one, joint post-entry profits are highest when one firm locates in the center and the other firm at the city limit—an asymmetry that has not arisen in the Hotelling model before. This dependence on $\theta$ is interesting in one more respect: When considering one dimension of product characteristics only, we implicitly assumed that products were perfectly homogeneous in all other dimensions. Now, they lost this property. In the quasi-linear utility model, the degree of differentiation as measured by $\theta$ affects decisions in the first dimension of product space (measured along the unit line). Even though $\theta$ does not add a further dimension in a strict sense, this result points towards a new array of possible interactions *across* product characteristics. A further step of the analysis could therefore be to consider the conjecture in a two-dimensional Hotelling model, where firms can choose location in a unit square, not only along the unit line.

# 3 The Two Models Compared and a View on the Literature

The circumstances under which it is more desirable for firms to allow entry and competition than to merge differ substantially between the two models considered. In the simple Hotelling model, merger will be preferable if demand is high relative to transportation cost. On the other hand, entry and competition is more profitable than a merger in the Hotelling world whenever demand is relatively low compared to transportation cost. The reason is that an entrant can exactly capture the demand that was lost when the monopoly was the only supplier. The results of this first model support the conjecture.

The conjecture, however, is essentially a claim in multi-dimensional characteristics space. Mergers tend to unify their products only in some dimensions and to leave them distinct in others. Merged car manufacturers may equip their cars with the same engine, but leave their cars' chassis strongly differentiated. This paper has therefore gone on to investigate the conjecture in a special model of two-dimensional product characteristics. In this model, products continue to be differentiated along the unit line à la Hotelling. But utility is quasi-linear and allows for the possibility that the two goods be substitutes (or complements) in a second dimension. In this model, a merger means a unification of the product characteristic measured along the unit line, whereas the goods remain differentiated in the second dimension. Merger generally dominates entry in this model unless the two goods become close substitutes in the second dimension. In addition, as the goods become close substitutes in the second dimension, the most profitable constellation is not where goods are maximally differentiated along the unit line. The most profitable constellation turns out be the one where one good is positioned at the center of the unit line and the second good at the boundary. The reason is the following: Selling one centrally positioned good reduces transportation cost for a maximal number of consumers and thus increases demand. If the other good happens to be positioned far from the center of the unit line (after entry), the negative effect of the goods' substitutability on profits is weak. The entrant and the incumbent do not steal much of each other's demand by selling their substitutes. This result is new. It might be specific to the assumptions of the second model. We could gain additional insight from carrying out a similar analysis in the framework of a pure Hotelling model with two dimensions.

There are two specific benefits to the second model: First, it explores a way to integrate the two principal modelling approaches of substitutablity as discussed at the outset of section 1. So far, Hotelling type models of linear or circular cities and models of 'fixed' substitutability have been largely unrelated. The model of the second section is a quick and simple attempt to combine quasi-linear utility with Hotelling elements. It allows to explore some consequences of such a combination. Much work remains to be done to reconcile the vastly different approaches to substitutability and product differentiation. It could be fruitful to try to develop a class of models that contain both the 'fixed' substitutability and the Hotelling based approaches as special cases. From such an integrated model we would gain a clearer idea of the theoretical implications that the different models of product differentiation implicitly have. Economists have started to compare vertical and horizontal differentiation and different Hotelling type models, but to my knowledge 'fixed' substitutability models and Hotelling type models remain largely unrelated.

Second, the model of the second section allows to consider partial complementarity, a puzzling topic to be investigated further. How can goods be complementary in one respect, and substitutes in another? Think of laptops and desktop computers. They are substitutes when it comes to writing documents or calculating economic models. They are imperfect substitutes because desktops usually have more comfortable screens and are cheaper to upgrade, but laptops are transportable. They are complementary because documents and files created on one can be transferred between the two computers, to whichever computer is more convenient to use at a time. In addition, when there are devices that can be used for both computers such as printers or external drives, buying a laptop in addition to a desktop becomes cheaper, and vice versa. Similar observations may be true for palm pilots and laptop computers.

The general question to ask is then: Can we device a nice space of functional forms for preferences that allows for both, simultaneous complementarities and substitutability? The most general form of utility functions incorporates this, of course. But in these functions, it is not clear at all what a bundle of goods is. In principle, one good is 'having a laptop available at noon', whereas another good is 'having a desktop at noon,' and still another good is 'having a laptop at 1pm.' Whenever we want to sensibly talk about goods and markets, we have some kind of aggregation across time and across characteristics in mind. A laptop is a laptop is a laptop at all times. A desktop is a substitute to a laptop at all times. And with only a small step in thought, we have irrevocably left behind the general utility model. Possibly, higher-dimensional spaces of product characteristics can accomodate more demanding concepts of goods without becoming too general to be useless. This is another, and much more fundamental topic that may deserve exploration.

# 4 Conclusion

This paper departed from a simple conjecture: *Given their product characteristics, firms will choose to enter and compete whenever their products are strongly differentiated, and to merge whenever their products are close in* *characteristics*. The models in the sections 1 and 2 have shown that it can be more profitable indeed to *enter and compete* than to *merge and match* products in their characteristics. The underlying reasoning is that a merger unifies the product characteristics in some respect, which in turn reduces demand. Under some circumstances, this effect can be so strong that profits of a merger are lower than the sum of profits after entry and competition. But then at least one firm will prefer not to merge, so that a merger will not occur. Entry and competition will follow.

The models rested on the observation that a merger will, in general, not only result in joint operations of the two merging firms but also in a matching of certain product characteristics. It is true that, in principle, a merger could have price collusion as its only objective. Then, product characteristics would remain untouched. However, the more recent economic literature on mergers has mostly focused on synergies or asset complementarities between the merging firms. As soon as a merger exploits synergies in production or R&D, however, products become more unified as a consequence. This is aspect of a merger has been mostly neglected so far. The models in this paper may also shed some light on the question why we have observed such a strong increase in mergers and associations across national borders lately. A possible explanation is that entry occurs when product space is still vast and largely unexplored. This may have been the case in many markets in previous decades. When product space becomes occupied, however, mergers start to be more attractive. The models of this paper lent support to this hypothesis by showing that entry occurs when products are far apart in their degree of differentiation.

# A Appendix

In order to prove proposition 1, a monopoly's (a merger's) price choice must be derived, given its location in product space, $\lambda_M$.

**Claim 1** *The optimal price choice of a merger is as depicted in table 1 (page 9).*

**Proof.** A consumer $k$ chooses to buy from the monopoly if $\bar{s} - (\lambda_M - x_k)^2 t - p_M > 0$, where $\bar{s}$ denotes a consumer's valuation of the good, $\lambda_M \in [0,1]$ the location of the monopoly along the unit line, measured from point zero, and $x_k \in [0,1]$ the location of consumer $k$. Given the monopoly price $p_M$, the first consumer at the left end to be indifferent between buying or not, lives at

$$\max\left[0, \lambda_M - \sqrt{\frac{\bar{s} - p_M}{t}}\right]$$

and the last consumer at the right end to be indifferent between buying or not, lives at

$$\min\left[0, \lambda_M + \sqrt{\frac{\bar{s} - p_M}{t}}\right].$$

Hence, the monopoly faces a demand of

$$\int_{\max\left[0,\lambda_M - \sqrt{\frac{\bar{s}-p_M}{t}}\right]}^{\lambda_M} f(x)dx + \int_{\lambda_M}^{\min\left[\lambda_M + \sqrt{\frac{\bar{s}-p_M}{t}},1\right]} f(x)dx,$$

where $f(x)$ is the density function of the consumer distribution over the unit line. For a uniform distribution, demand simplifies to

$$D_M = \min\left[\lambda_M + \sqrt{\frac{\bar{s} - p_M}{t}}, 1\right] - \max\left[0, \lambda_M - \sqrt{\frac{\bar{s} - p_M}{t}}\right]. \tag{19}$$

Still, the profit function is kinked and thus not differentiable at $p_M = \bar{s} - (1-\lambda_M)^2 t$ and $p_M = \bar{s} - \lambda_M^2 t$. For $\lambda_M = \frac{1}{2}$, these two kinks coincide. As it turns out, three ranges of price choice (and some subcases) have to be distinguished for $\lambda_M \neq \frac{1}{2}$ (because demand can be lost on two sides of the unit line, on one side, or not at all) and two price ranges for $\lambda_M = \frac{1}{2}$ (because either demand is lost on both sides of the unit line or not at all). The merged firm maximizes monopoly profits

$$\max_{p_M} \Pi_M = (p_M - c)\left( \min\left[\lambda_M + \sqrt{\frac{\bar{s} - p_M}{t}}, 1\right] \right.$$
$$\left. - \max\left[0, \lambda_M - \sqrt{\frac{\bar{s} - p_M}{t}}\right] \right). \quad (20)$$

Without loss of generality, suppose that the monopoly is located at $\lambda_M < \frac{1}{2}$ for the derivation of the aforementioned three ranges. First, consider price range (1) where monopoly price is so low that all consumers on the unit line buy. Thus, $p_M \leq \bar{s} - (1 - \lambda_M)^2 t < \bar{s} - \lambda_M^2 t$ (for $\lambda_M < \frac{1}{2}$) or, equivalently, $1 - \lambda_M \leq \sqrt{\frac{\bar{s} - p_M}{t}}$. Then the monopoly maximizes profits

$$\Pi_M = (p_M - c) \cdot 1 \qquad \text{s.t.} \quad p_M \leq \bar{s} - (1 - \lambda_M)^2 t.$$

The optimal price simply lies at the constraint, $p_M^1 = \bar{s} - (1 - \lambda_M)^2 t$, and maximal profits are $\Pi_M^1 = \bar{s} - c - (1 - \lambda_M)^2 t$. (The second order condition can be shown to be satisfied.) Consequently, $\frac{\bar{s} - c}{t} < 3\lambda_M^2$ in the first range (compare the first row of table 1, p. 9).

Second, consider the intermediate range (2), where monopoly price is low enough that all consumers on the left side of $\lambda_M < \frac{1}{2}$ buy, but some on the right side do not buy. Thus, $\bar{s} - (1 - \lambda_M)^2 t < p_M \leq \bar{s} - \lambda_M^2 t$ or, equivalently, $1 - \lambda_M > \sqrt{\frac{\bar{s} - p_M}{t}}$ and $\lambda_M \leq \sqrt{\frac{\bar{s} - p_M}{t}}$ (for $\lambda_M < \frac{1}{2}$). Then the monopoly maximizes profits

$$\Pi_M = (p_M - c)\left(\lambda_M + \sqrt{\frac{\bar{s} - p_M}{t}}\right) \quad \text{s.t. } p_M \in \left(\bar{s} - (1 - \lambda_M)^2 t, \bar{s} - \lambda_M^2 t\right]. \quad (21)$$

If it exists, the optimal *interior* price choice is

$$p_M^2 = \frac{2\bar{s} + c}{3} - \frac{2}{3} t \lambda_M \left(\frac{\lambda_M}{3} \pm \sqrt{3\frac{\bar{s} - c}{t} + \lambda_M^2}\right), \quad (22)$$

under the first order condition

$$\frac{\partial \Pi_M}{\partial p_M} = \lambda_M + \frac{3(\bar{s} - p_M) - (\bar{s} - c)}{2\sqrt{t(\bar{s} - p_M)}}$$

and the second order condition

$$\frac{\partial^2 \Pi_M}{\partial p_M^2} = -\frac{3(\bar{s}-p_M)+(\bar{s}-c)}{4(\bar{s}-p_M)\sqrt{t(\bar{s}-p_M)}} < 0 \qquad \text{for} \quad p_M \leq \bar{s}-\lambda_M^2 t.$$

I.e., profits are increasing in $p_M$ irrespective of $\lambda_M$. Now, three subcases have to be considered. Observe that, if the first order condition has the same sign at both the lower *and* upper price boundary, $\bar{s}-(1-\lambda_M)^2 t$ and $\bar{s}-\lambda_M^2 t$, then an interior solution cannot exist. Evaluating the first order condition at the lower price bound yields

$$\begin{aligned} \left.\frac{\partial \Pi_M}{\partial p_M}\right|_{p_M=\bar{s}-(1-\lambda_M)^2 t} &= \frac{t(1-\lambda_M)(3-\lambda_M)-(\bar{s}-c)}{2t(1-\lambda_M)} \geq 0 \\ &\iff \quad \frac{\bar{s}-c}{t} \leq (1-\lambda_M)(3-\lambda_M), \end{aligned}$$

whereas evaluation at the upper price bound yields

$$\left.\frac{\partial \Pi_M}{\partial p_M}\right|_{p_M=\bar{s}-\lambda_M^2 t} = \frac{5t\lambda_M^2-(\bar{s}-c)}{2t\lambda_M} \geq 0 \quad \iff \quad \frac{\bar{s}-c}{t} \leq 5\lambda_M^2.$$

The three subcases to distinguish are: First (2a), $\frac{\bar{s}-c}{t} \leq 5\lambda_M^2 < (1-\lambda_M)(3-\lambda_M)$, in which case $p_M^2$ will be at the upper bound. Second (2b), $5\lambda_M^2 < \frac{\bar{s}-c}{t} < (1-\lambda_M)(3-\lambda_M)$ for the above interior solution. And third (2c), $5\lambda_M^2 < (1-\lambda_M)(3-\lambda_M) \leq \frac{\bar{s}-c}{t}$, in which case $p_M^2$ will be at the lower bound. Table 1 (p. 9) shows the corresponding price choices and maximal profits. Note that only the smaller root in (22) leads to consistent subcases at the bounds of the price ranges. The optimal price choices can be shown to be equal to each other at the bounds by deriving the two or four respective roots that solve the equalities for $\lambda_M$ and picking the root that coincides with the respective bound of $\frac{\bar{s}-c}{t}$.

Third, consider the other extreme range (3) where monopoly price is so high that demand is lost on both sides of the unit line. Thus, $\bar{s} \geq p_M > \bar{s}-\lambda_M^2 t$ or, equivalently, $\lambda_M > \sqrt{\frac{\bar{s}-p_M}{t}}$ (for $\lambda_M < \frac{1}{2}$). Then the optimal price is $p_M^3 = \arg\max (p_M - c) 2\sqrt{\frac{\bar{s}-p_M}{t}} = \frac{2\bar{s}+c}{3}$ and maximal profits are $\Pi_M^3 = 4t\left(\frac{\bar{s}-c}{3t}\right)^{\frac{3}{2}}$, irrespective of $\lambda_M$. Note that this case can never arise for $\lambda_M = 0$ since the stated range conditions for $\lambda_M$ and the optimal price choice imply $\sqrt{\frac{\bar{s}-p_M}{3t}} < \lambda_M$.

A similar derivation can be done for $\lambda_M > \frac{1}{2}$. In the case of $\lambda_M = \frac{1}{2}$, subcase (2b) is not well defined, and ranges (1) and (2) coincide. ■

As can be seen from column 4 in table 1 (p. 9), profits increase in $\lambda_M$. Therefore, the merger will pick $\lambda_M = \max[\lambda_I, \lambda_E]$ in optimum. In order to prove proposition 1, take the differences between joint profits after merger and under entry for each of the 4 ranges in table 1:

$$\begin{aligned} & (\Pi_M^n - F_M) - (\Pi_E + \Pi_I - F_E) \\ = \; & \Pi_M^n + (F_E - F_M) - t\left[1 + \left(\frac{\lambda_I - \lambda_E}{3}\right)^2\right][(1-\lambda_E) - \lambda_I], \end{aligned}$$

where $n$ denotes the number of the range.

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