Lecture Notes, January 11, 2010

Partial equilibrium comparative statics

Partial equilibrium: Market for one good only with supply and demand as a function of price. Price is defined as the solution to the equation.

$$z(p) = D(p) - S(p) = 0$$

The implicit assumption is *ceteris paribus*, other things being equal (all other prices held fixed). Suppose there is a shift parameter, α, that describes changes in the demand and supply functions. Then the definition of equilibrium now looks like:

$$z(p,\alpha) = D(p,\alpha) - S(p,\alpha) = 0.$$

Consider changes in α. What happens to p?

Totally differentiate z with respect to α. We have

$$\frac{dz}{d\alpha} = \frac{\partial z}{\partial p}\frac{dp}{d\alpha} + \frac{\partial z}{\partial \alpha} = 0$$

Assuming $\frac{\partial z}{\partial p} \neq 0$ we have

$$\frac{dp}{d\alpha} = -\left(\frac{1}{\frac{\partial z}{\partial p}}\right)\frac{\partial z}{\partial \alpha} = -\frac{\frac{\partial z}{\partial \alpha}}{\frac{\partial z}{\partial p}} = -\frac{D_\alpha - S_\alpha}{D_p - S_p}$$

(the denominator $\frac{\partial z}{\partial p} = D_p - S_p$ of this expression is the Jacobian of the system).

Then suppose that α represents an upward shift in demand and that the usual slopes apply to D and S. $D_p < 0, S_p > 0$. We have

$$\frac{dp}{d\alpha} = -\frac{(+)-(0)}{(-)-(+)} = -\left(\frac{+}{-}\right) = +$$

Just what you'd expect. An upward shift in demand increases price.

Example: The tax incidence problem

Who really pays a tax levied on sellers?

Let α = excise tax, $p^o - \alpha$= price received by seller, p^o = price paid by buyer

$D(p, \alpha) = D(p, 0)$, $S(p, \alpha) = S(p - \alpha, 0)$

$D_\alpha(p, \alpha) = 0$, $S_\alpha(p, \alpha) = - S_p$.

$$\frac{dp^o}{d\alpha} = -\frac{D_\alpha - S_\alpha}{D_p - S_p} = -\frac{-(-S_p)}{D_p - S_p} = \frac{-S_p}{D_p - S_p}$$

Consider the case $S_p >> 0$, $D_p \approx 0$; elastic supply, inelastic demand.

Then $\frac{dp^o}{d\alpha} \approx 1$. Interpretation: Price to seller is unaffected by imposition of the tax α . The tax is shifted to buyers.

Comparative Statics, Implicit Function Theorem

Characterize market equilibrium, subject to a parameter α, by market clearing in $z(p, \alpha)$.

$$\begin{pmatrix} z_1(p,\alpha) \\ \vdots \\ z_i(p,\alpha) \\ \vdots \\ z_N(p,\alpha) \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Prices p are endogenously determined by the market clearing condition. Then as α shifts, market-clearing values of p will change as well. Assuming everything in sight is differentiable and well defined, we have,

$$\begin{pmatrix} \frac{dz_1}{d\alpha} \\ \vdots \\ \vdots \\ \vdots \\ \frac{dz_N}{d\alpha} \end{pmatrix} = \begin{pmatrix} \frac{\partial z_1}{\partial p_1} & & & \frac{\partial z_1}{\partial p_N} \\ & \ddots & & \\ \vdots & & \frac{\partial z_i}{\partial p_j} & \vdots \\ & & \ddots & \\ \frac{\partial z_N}{\partial p_1} & & & \frac{\partial z_N}{\partial p_N} \end{pmatrix} \begin{pmatrix} \frac{dp_1}{d\alpha} \\ \vdots \\ \frac{dp_i}{d\alpha} \\ \vdots \\ \frac{dp_N}{d\alpha} \end{pmatrix} + \begin{pmatrix} \frac{\partial z_1}{\partial \alpha} \\ \vdots \\ \frac{\partial z_i}{\partial \alpha} \\ \vdots \\ \frac{\partial z_N}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The expression $\begin{pmatrix} \frac{\partial z_1}{\partial p_1} & & & \frac{\partial z_1}{\partial p_N} \\ & \ddots & & \\ \vdots & & \frac{\partial z_i}{\partial p_j} & \vdots \\ & & \ddots & \\ \frac{\partial z_N}{\partial p_1} & & & \frac{\partial z_N}{\partial p_N} \end{pmatrix}$ is the ***Jacobian*** of the market clearing equation system. Solving for $\begin{pmatrix} \frac{dp_1}{d\alpha} \\ \vdots \\ \frac{dp_i}{d\alpha} \\ \vdots \\ \frac{dp_N}{d\alpha} \end{pmatrix}$ we have

$$\begin{pmatrix} \frac{dp_1}{d\alpha} \\ \vdots \\ \frac{dp_i}{d\alpha} \\ \vdots \\ \frac{dp_N}{d\alpha} \end{pmatrix} = -\begin{pmatrix} \frac{\partial z_1}{\partial p_1} & & & \frac{\partial z_1}{\partial p_N} \\ & \ddots & & \\ \vdots & & \frac{\partial z_i}{\partial p_j} & \vdots \\ & & \ddots & \\ \frac{\partial z_N}{\partial p_1} & & & \frac{\partial z_N}{\partial p_N} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial z_1}{\partial \alpha} \\ \vdots \\ \frac{\partial z_i}{\partial \alpha} \\ \vdots \\ \frac{\partial z_N}{\partial \alpha} \end{pmatrix}$$

This expression is well defined when the Jacobian is non-singular. This is an application of the **Implicit Function Theorem.**

See also **Regular Economies**.

HELP: There is a shortage of good, intelligent, relatively simple transparent, comparative statics problems suitable for a problem set or exam. Please suggest your favorite to Ross. Reward: 3 brownie points plus your question may show up where it will do you the most good.

Consumer Surplus and Compensation Tests

Mas-Colell notation, ch. 10.

What we say in Econ 1: Competitive Equilibrium optimizes triangle area of total surplus.

Du Puit: Ecole des Ponts et Chaussées
Valuing a bridge across the Seine

Embarrassing variety of consumer surplus measures
equivalent variation; compensating variation
resulting from income effects.

MasColell & Alfred Marshall: Assume negligible income effects and that marginal utility of income is constant. This implies validity of partial equilibrium (*ceteris paribus* --- other things being equal) treatment.

Results to be demonstrated:
Proposition: 1. Welfare optimization (Pareto efficiency subject to income redistribution) is equivalent to maximizing
Marshallian Surplus = Consumer Surplus + Producer Surplus
= Consumer Surplus + Profits
2. (1FTWE) Competitive Equilibrium allocation is Pareto efficient (Marshallian Surplus maximizing).
3. (2FTWE) Any Pareto efficient allocation can be supported as a competitive equilibrium, subject to a redistribution of income.

Model:
$i \in H$, $j \in F$
good m = Hicksian composite of all goods but one with prices held constant (partial equilibrium, *ceteris paribus* = other things being equal)
m is numeraire, price set equal to unity, 1.
good ℓ, price of good ℓ is p, market determined

Production
$c^j(q)$ = firm j's cost function

= j's input requirement (in m) to produce q of ℓ

q^j = output of firm j

Households

m^i = i's consumption of m

x^i = i's consumption of ℓ

ω^i = i's endowment of m,

u^i = i's utility function = $m^i + \varphi^i(x^i)$

quasi-linearity, partial equilibrium, constant marginal utility of income (this is equivalent to assuming "other things being equal", all prices except ℓ's held constant; implying constant marginal rates of substitution across all goods other than ℓ, hence valid aggregation).

Firms

Profit of firm j at price p is defined as $\pi^j = p \cdot q^j - c^j(q^j)$

$1 \geq \theta^{ij} \geq 0$, θ^{ij} is i's ownership share of firm j, $\sum_{i \in H} \theta^{ij} = 1$

Competitive equilibrium

p^o, x^{io}, q^{jo} so that

$p^o = c^{j\prime}(q^{jo})$, all j, $p^o = \varphi^{i\prime}(x^{io})$, all i,

(income conditions) $p^o \cdot x^{io} + m^i = \omega^i + \sum_{j \in F} \theta^{ij} \pi^j$, all i, and

$$\sum_{i \in H} x^i = \sum_{j \in F} q^j \quad \text{(market clearing).}$$

Determination of the (efficient/equilibrium) quantity of good ℓ in MasColell, Whinston & Green's quasi-linear model

The only thing that determines the gross quantity of ℓ in this model is the first order condition $\varphi^{i\prime}(x^i) = c^{j\prime}(q^j)$ for all i in H, all j in F, (assuming interior solution for x^i, q^j). There is no effect from the total endowment of m, $\sum_{i \in H} \omega^i$. The reason for this is that we purposely omit any nonnegativity condition on m. Thus total m used as inputs for producing ℓ may be more than total endowment. If that happens some households end up with large negative holdings of m.

The initial endowment ω^i is very important in determining the competitive equilibrium distribution of welfare --- since it represents initial wealth, but it has no effect on the equilibrium quantities of ℓ held individually, x^i.

This is a massively oversimplified model. The purpose is to emphasize the notion of the relation of competitive equilibrium and efficiency to consumer and producer surplus. It does that effectively at the cost of great oversimplification.

Welfare Economics

The quasi-linear form of u^i makes welfare economics very simple. Note that any attainable plan will have the property that $\sum_{i\in H} x^i = \sum_{j\in F} q^j$. The linear form of u^i says that the utility possibility frontier is a straight line. Choose q^j efficiently and then distribute resulting ℓ to max sum of $\varphi^i(x^i)$, then redistribute ω^i for desired mix of utilities.

Any attainable Pareto efficient allocation of resources and consumption (ignoring boundary conditions) is characterized as choosing x^i, q^j, so that, $\sum_{i\in H} x^i = \sum_{j\in F} q^j$, to maximize

$$\sum_{i\in H} u^i = \sum_{i\in H} [m^i + \varphi^i(x^i)]$$

$$= \sum_{i\in H} [\varphi^i(x^i) + \omega^i - px^i + (\sum_{j\in F} \theta^{ij}\pi^j)]$$

$$= \sum_{i\in H} [\varphi^i(x^i) + \omega^i - px^i + \{\sum_{j\in F} \theta^{ij}(p\, q^j - c^j(q^j)\}]$$

$$= \sum_{i\in H} \varphi^i(x^i) - \sum_{i\in H} px^i + \sum_{i\in H} \omega^i$$

$$+\sum_{i\in H}\left(\sum_{j\in F}\theta^{ij}pq^j\right)-\sum_{i\in H}\left(\sum_{j\in F}\theta^{ij}c^j(q^j)\right)$$

$$=\sum_{i\in H}\varphi^i(x^i)-\sum_{i\in H}px^i+\sum_{i\in H}\omega^i+\sum_{j\in F}pq^j-\sum_{j\in F}c^j(q^j)$$

= Consumer surplus + endowment + profit

$$=\sum_{i\in H}\varphi^i(x^i)+\sum_{i\in H}\omega^i-\sum_{j\in F}c^j(q^j)$$

But $\sum_{i\in H}\omega^i$ is a constant, so maximizing $\sum_{i\in H}u^i$ implies maximizing

$$\sum_{i\in H}\varphi^i(x^i)-\sum_{j\in F}c^j(q^j)$$

= consumer surplus + producer surplus = Marshallian surplus.

<u>Welfare Maximization in quasi-linear model:</u>

Maximize $S(x^1, x^2, ..., x^{\#H}; q^1, ..., q^{\#F})$

$$=\sum_{i\in H}\varphi^i(x^i)-\sum_{j\in F}c^j(q^j)$$

subject to $\sum_{i\in H}x^i=\sum_{j\in F}q^j$

$$\mathbf{L}=\sum_{i\in H}\varphi^i(x^i)-\sum_{j\in F}c^j(q^j)-\lambda\left(\sum_{i\in H}x^i-\sum_{j\in F}q^j\right)$$

$$\frac{\partial L}{\partial x^i}=\varphi^{i\prime}-\lambda=0$$

$$\frac{\partial L}{\partial q^i}=-c^{j\prime}+\lambda=0$$

Therefore the First Order Condition for Pareto Efficiency is

$$\boldsymbol{\varphi^{i\prime}=c^{j\prime}}$$

<u>First Fundamental Theorem of Welfare Economics in quasi-linear model</u>:

$\boldsymbol{\varphi^{i\,\prime} = c^{j\prime}} = p^o$ is the characterization of competitive equilibrium so Competitive Equilibrium is Pareto Effcient.

Second Fundamental Theorem of Welfare Economics: Any attainable Pareto efficient allocation can be sustained as a competitive equilibrium, $\boldsymbol{\varphi^{i\,\prime} = c^{j\prime}} = p^o$, subject to a redistribution of ω^i .

Compensation tests for public works:

- Pareto preferability
- Increase in Marshallian surplus (possibly without compensation) .

Note theory of the second best in the presence of distortionary taxation, Auerbach.