A Note on Indivisibilities, Specialization, and Economies of Scale

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It is well known that factor indivisibilities and opportunities for labor specialization (division of labor) can result in scale economies. We argue that the second observation is a special case of the first; labor specialization results in scale economies only through indivisibility or other nonconvexity in the use of labor. Hence, the observation that labor specialization results in scale economies is correct but a half-truth; it relies on the unstated assumption of indivisibility or nonconvexity in the use of labor.[1] Tjalling Koopmans (1957) citing E. H. Chamberlin (1948) and Nicholas Kaldor (1934), described this observation,

> The relevant aspect of worker specialization appears to be that, up to a certain degree of specialization, the undivided attention given by a specialized worker to a full-time task of a sufficiently challenging character produces not exactly (but presumably more than) twice as much as half-time attention (with half the training!) given to the same task, if the other half of the worker's time (and training) is applied to a different productive activity.
> [p. 151]

Adam Smith noted that "division of labor is limited by the extent of the market." On the contrary, if labor were fully divisible, Smith's statement would be false; there would be no particular reason why market size should pose a limitation on division of production tasks. If, however, labor is indivisible or displays nonconvexity in use, then Smith's statement is correct. Sufficient scale would be required to overcome indivisibilities to allow (indivisible) labor to specialize in separate portions of the production process. Alternatively, a setup cost (a nonconvexity) in the transition of labor between production operations is a sufficient condition for scale to be required to reduce average cost, through reduction of frequency of switching operations. Finally, if a setup cost in training time is needed for acquisition of a specialized skill, this nonconvexity will account for a scale economy in the employment of specialized labor for the production sector (but not necessarily for the individual firm).

I. The Pin Factor Example

In his pin factory analysis, Smith recognized the role of nonconvexity in labor use, attributing much of "the great increase in the quantity of work...in consequence of the division of labor...to the saving of the time which is commonly lost in passing from one specie of work to another" (p. 7). Hence, in Smith's view, employing the same worker at different tasks requires incurring a transition setup cost. Given sufficient scale, it is preferable to allow labor to specialize and avoid this switching cost, that is, to use labor in indivisible increments.

The production of pins will involve choosing one of many possible techniques of production. The crudest technology will involve using the same worker in all operations of

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[1] Indivisibilities need not be the sole rationale for scale economies in capital. Nicholas Kaldor (1972) attributes scale economies in part to the "three-dimensional nature of space." That is, that production capacity in some processes will vary with physical volume of plant or equipment while cost may depend principally on surface area, the latter varying as the two-thirds power of volume.

production. More sophisticated means of production are defined by having each unit of labor assigned to fewer operations.

> One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head; to make a head requires two or three distinct operations: to put it on, is a peculiar business, to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, which, in some manufactories, are all performed by distinct hands, though in others the same man will sometimes perform two or three of them. [Smith, pp. 4–5]

As an illustration of this example, consider a family of production functions, indexed by $k=1,\dots,n$, by which a single output, y, is produced. Although actual production of y will involve only one of the functions, progressively higher output levels will be achievable for a given set of inputs by using a more specialized production function. The limitation on specialization will be indivisibility or other nonconvexity, so that higher levels of specialization will be available only with sufficient input units. Consider the primary production function, defined by

$$(1) \qquad y=f_k(x_1,\dots,x_k)=b_k\prod_{i=1}^{k}x_i^{\alpha_{ik}}$$

where $y=$ output under process $k=1,\dots,n$; where $x_i=$ quantity of labor pursuing specialty i, $i=1,\dots,k$; for each k, $\sum_{i=1}^{k}\alpha_{ik}=1$, $\alpha_{ik}\geq 0$; where $b_k=$ technology parameter for process k; and $b_{k+1}>((k+1)/k)b_k$.

We have J workers, $j=1,\dots,J$. The variable x_{ij} is the amount of labor in specialty i provided by worker j: $x_i=\sum_{j=1}^{J}x_{ij}$. According to (1) there are n possible separate production processes, ranging from the simplest involving production by using only one task, ($k=1$, i.e., one class of labor input), to more complex ones involving many.

To convert f_k from constant returns to increasing returns let

$$(2) \quad F_k(x_{11},\dots,x_{ij},\dots,x_{kJ}) = f_k\left(\sum_{j=1}^{J}[x_{1j}],\dots,\sum_{j=1}^{J}[x_{kj}]\right),$$

where $[x_{ij}]$ denotes the greatest integer $\leq x_{ij}$; or let

$$(3) \quad G_k(x_{11},\dots,x_{ij},\dots,x_{kJ}) = f_k(x_1,\dots,x_k)-\sum_{i=1}^{k}\sum_{j=1}^{J}c_i\{x_{ij}\},$$

where $c_i>0$ and $\{x_{ij}\}$ is defined to be

$$0 \quad \text{for } x_{ij}=0 \qquad \text{and} \qquad 1 \quad \text{for } x_{ij}>0.$$

To represent the sources of scale economy, we consider indivisibility (2), and setup cost (3). The advantages of specialization are embodied in the assumption that $b_{j+1}>((j+1)/j)b_j$. As a result, the production functions defined in equations (2) and (3) have the following characteristics: 1) for a given value of labor input, $\sum_i\sum_j x_{ij}$, production has higher average product (value of f_k) as we move from lower-order to higher-order processes within limits imposed by indivisibility and setup cost; 2) higher-order processes involve a finer division of inputs; and 3) while production within each process is characterized by constant returns to scale, the presence of indivisibilities or setup costs in the use of inputs results (though not uniformly) in increasing returns to scale.

II. Labor Specialization and Scale Economies in the Elementary Literature

Indivisibilities in equipment and the desirability of labor specialization are cited in the elementary literature as distinct sources of scale economies in production. Indivisibility (or other nonconvexity) of labor is seldom made explicit. Edwin Mansfield's text (1976) is typical of treatments that use labor specialization as a rationale for scale economies without treating indivisibility of labor as a logical step:[2]

> Some inputs are not available in small units; for example, we cannot install half an open hearth furnace. Because of indivisibilities of this sort, increasing returns to scale may occur.... Greater specialization also can result in increasing returns to scale; as more men and machines are used, it is possible to subdivide tasks and allow various inputs to specialize.
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> [Mansfield, pp. 128, 129]

III. Conclusion

The possibility of productively superior specialization of labor is not, in itself, a sufficient condition for the presence of scale economies in production. The link between specialization (division of labor) and scale economies is indivisibility or other nonconvexity in application of labor. The classic treatment of Smith implicitly recognized this point and it was elaborated by Koopmans. Nevertheless, it is not explicit in the current elementary literature, which thereby obscures the logic of the analysis.

[2]The following standard texts recognize scale economies without going into detail as to their relationship to labor specialization: Charles Baird (1975), James Gwartney and Richard Stroup (1980), David Kamerschen and Lloyd Valentine (1981), E. Warren Shows and Robert Burton (1972). Alternatively, Stanley Kaish (1976), Paul Samuelson (1980), and Donald Watson and Malcolm Getz (1981) treat specialization and scale in a fashion similar to Mansfield. Richard Lipsey and Peter Steiner (1975) is an exception in explicitly recognizing indivisibility in use of labor. Joan Robinson and John Eatwell (1973) emphasize organizational and distributional issues rather than technology in their discussion of division of labor.

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