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Midterm 1

This exam is open book, open notes. Other people are closed. **Answer any TWO problems from Part I and any TWO from Part II.** They count equally.

PART I, Answer two problems

Problems 1, 2, and **3** work with a two-person pure exchange economy (an Edgeworth Box). Let there be two households with different endowments. Superscripts are used to denote the name of the households. There are two commodities, x and y. For simplicity, let the two households each have the same tastes (same form of the utility function). Household 1 is characterized as $u^1(x^1, y^1) = x^1y^1$, with endowment

$r^1 = (8, 0)$.

Note that 1's MRS at (x^1, y^1) can be characterized (assuming positive values of x^1, y^1) as

$$MRS^1_{xy} = \frac{\frac{\partial u^1}{\partial x}}{\frac{\partial u^1}{\partial y}} = \frac{y^1}{x^1}$$

Household 2 is characterized as

$u^2(x^2, y^2) = x^2y^2$, with endowment

$r^2 = (2, 10)$.

(The superscripts are household names, not powers) Note that 2's MRS at (x^2, y^2) can be characterized (assuming positive values of x^2, y^2) as

$$MRS^2_{xy} = \frac{\frac{\partial u^2}{\partial x}}{\frac{\partial u^2}{\partial y}} = \frac{y^2}{x^2}$$

Recall that when a household optimizes utility subject to budget constraint at prices (p_x, p_y) it chooses x, y so that

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$MRS_{xy} = \frac{p_x}{p_y}$ and so that $p_x x + p_y y$ = the household's budget = value of household endowment at (p_x, p_y).

A competitive equilibrium consists of prices $p^o = (p^o_x, p^o_y)$ and allocation $(x^{o1}, y^{o1}), (x^{o2}, y^{o2})$ so that

(a) household 1's consumption plan (x^{o1}, y^{o1}) maximizes $u^1(x, y)$ subject to household 1's budget constraint, $p^o_x x + p^o_y y = 8p^o_x$, and similarly

(b) household 2's consumption plan (x^{o2}, y^{o2}) maximizes 2's utility subject to 2's budget, $p^o_x x + p^o_y y = 2p^o_x + 10p^o_y$ and

(c) markets clear: $(x^{o1}, y^{o1}) + (x^{o2}, y^{o2}) = (8,0) + (2,10) = (10, 10)$.

Let prices be $(p_x, p_y)=(1/2 , 1/2)$. Then household 1's utility maximizing plan subject to budget constraint is $(x^{o1}, y^{o1}) = (4,4)$ and household 2's utility maximizing plan subject to budget constraint is $(x^{o2}, y^{o2}) = (6,6)$

1. Is the price vector $(p^o_x, p^o_y)=(1/2 , 1/2)$ a competitive equilibrium? Explain.

2. Demonstrate that, at the allocation $(x^{o1}, y^{o1}) = (4,4)$, $(x^{o2}, y^{o2}) = (6,6)$, we have $MRS^1_{xy} = MRS^2_{xy}$. This is sufficient to show that the allocation is Pareto efficient.

3. When the price system finds prices that clear the market, (c), the prices are said to 'decentralize' the equilibrium allocation. Explain this notion of 'decentralize' or 'decentralization.'

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PART II, Answer two problems

Recall the following definitions, concerning subsets of R^N:

- a set is 'closed' if it contains all of its cluster points (limit points).
- a set is 'open' if, for each point in the set, there is a small ball (neighborhood) centered at the point, contained in the set.
- a set is 'bounded' if it can be contained in a cube of finite size, centered at the origin.
- a set is 'compact' if it is both closed and bounded.
- a set is 'convex' if for every two points in the set, the set includes the line segment connecting them.

5. (i) Is the following subset of R^2 closed? open? bounded? compact? convex? Explain your answer.

T = 45^o line through the origin = $\{(x,y)| (x,y) \in R^2, x=y\}$

(ii) Is the following subset of R^2 closed? open? bounded? compact? convex? Explain your answer.

U = ball of radius 10 centered at the origin, not including its boundary = $\{(x,y)| (x,y) \in R^2, x^2+y^2 < 100\}$

6. Consider the following functions from R into R.

(a) $f(x) = x^2$. Is f continuous at 0? Explain your answer (a non-technical explanation is sufficient, you don't need to do an ε–δ proof).

(b) $g(x) = 0$ for $-1 \le x \le 1$,
$g(x) = 1$ for $x< -1$ and for $x >1$.

Is g continuous at x = -1? Explain your answer. Is g continuous at x = 0? Explain your answer. (Non-technical explanations are sufficient).

7. The Brouwer Fixed Point Theorem says that if S is a compact convex subset of R^N and if f is continuous, $f:S\rightarrow S$, then there is $x^* \in S$ so that $f(x^*) = x^*$; x^* is a fixed-point of the mapping f. For the following combinations of f and S, does f have a fixed point? Explain your answer.

(i) S = R (the real line), $f(x) = x + 1$.

(ii) $B = \{(x,y) \in R^2 \mid x^2+y^2 \le 100\}$ ball of radius 10 centered at the origin, $f(x,y) = -(x,y)$. f maps each point of the ball to its diametric opposite point.