Lecture Notes, Lecture 3

The Edgeworth Box

2 person, 2 good, pure exchange economy

Fixed positive quantities of X and Y, and two households, 1 and 2.

Household 1 is endowed with $\overline{X}^1$ of good X and $\overline{Y}^1$ of good Y, utility function $U^1(X^1, Y^1)$. Household 2 is endowed with $\overline{X}^2$ of good X and $\overline{Y}^2$ of good Y, utility function $U^2(X^2, Y^2)$

$$X^1 + X^2 = \overline{X}^1 + \overline{X}^2 = \overline{X},$$
$$Y^1 + Y^2 = \overline{Y}^1 + \overline{Y}^2 = \overline{Y}.$$

Each point in the Edgeworth box represents an attainable choice of X^1 and X^2 , Y^1 and Y^2 .
1's origin is at the southwest corner; 1's consumption increases as the allocation point moves in a northeast direction; 2's increases as the allocation point moves in a southwest direction. Superimpose indifference curves on the Edgeworth Box.

<u>Pareto efficiency:</u>

An allocation is Pareto efficient if all of the opportunities for mutually desirable reallocation have been fully used. The allocation is Pareto efficient if there is no available reallocation that can improve the utility level of one household while not reducing the utility of any household.

Tangency of 1 and 2's indifference curves : Pareto efficient allocations.

Pareto efficient allocation:
$(X^{o1}, Y^{o1}), (X^{o2}, Y^{o2})$ maximizes

$U^1(X^1, Y^1)$ subject to $U^2(X^2, Y^2) = U^2(X^{o2}, Y^{o2}) \equiv U^{o2}$, and subject to the resource constraints

$$X^1 + X^2 = \overline{X}^1 + \overline{X}^2 = \overline{X}$$

$$Y^1 + Y^2 = \overline{Y}^1 + \overline{Y}^2 = \overline{Y} \quad .$$

Equivalently, $X^2 = \overline{X} - X^1$, $\quad Y^2 = \overline{Y} - Y^1$

Lagrangian

$$L \equiv U^1(X^1, Y^1) + \lambda[U^2(\overline{X} - X^1, \overline{Y} - Y^1) - U^{o2}]$$

$$\frac{\partial L}{\partial X^1} = \frac{\partial U^1}{\partial X^1} - \lambda \frac{\partial U^2}{\partial X^2} = 0$$

$$\frac{\partial L}{\partial Y^1} = \frac{\partial U^1}{\partial Y^1} - \lambda \frac{\partial U^2}{\partial Y^2} = 0$$

$$\frac{\partial L}{\partial \lambda} = U^2(X^2, Y^2) - U^{o2} = 0$$

This gives us then the condition

$$MRS^1_{xy} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = MRS^2_{xy} \quad \text{or equivalently}$$

$$MRS^1_{xy} = -\frac{\partial Y^1}{\partial X^1}\Big|_{U^1=\text{constant}} = -\frac{\partial Y^2}{\partial X^2}\Big|_{U^2=\text{constant}} = MRS^2_{xy}$$

Pareto efficient allocation in the Edgeworth box: the slope of 2's indifference curve at an efficient allocation will equal the slope of 1's indifference curve; the points of tangency of the two curves.

contract curve = individually rational Pareto efficient points

<u>Market allocation</u>

p^x, p^y

Household 1: Choose X^1, Y^1, to maximize $U^1(X^1,Y^1)$ subject to

$$p^x X^1 + p^y Y^1 = p^x \overline{X}^1 + p^y \overline{Y}^1 = B^1$$

budget constraint is a straight line passing through the endowment point $(\overline{X}^1, \overline{Y}^1)$ (equivalently $(\overline{X}^2, \overline{Y}^2)$) with slope $-(p_x/p_y)$.

Lagrangian

$L = U^1(X^1,Y^1) - \lambda\, [p^x X^1 + p^y Y^1 - B^1]$

$$\frac{\partial L}{\partial X} = \frac{\partial U^1}{\partial X^1} - \lambda p^x = 0$$

$$\frac{\partial L}{\partial Y} = \frac{\partial U^1}{\partial Y^1} - \lambda p^y = 0$$

Therefore, at the utility optimum subject to budget constraint we have

$$MRS^1_{xy} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = \frac{p^x}{p^y}$$; Similarly for household 2,

$$MRS^2_{xy} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = \frac{p^x}{p^y} \ .$$

Equilibrium prices: p^{*x} and p^{*y} so that

$$X^{*1} + X^{*2} = \overline{X}^1 + \overline{X}^2 = \overline{X}$$

$$Y^{*1} + Y^{*2} = \overline{Y}^1 + \overline{Y}^2 = \overline{Y},$$

where X^{*i} and Y^{*i} , i =1, 2, are utility maximizing mix of X and Y at prices p^{*x} and p^{*y} .

Market clearing

$$\frac{p^x}{p^y} = \frac{\frac{\partial U^1}{\partial X^1}}{\frac{\partial U^1}{\partial Y^1}} = -\frac{\partial Y^1}{\partial X^1}\Big|_{U^1=U^{1*}} = -\frac{\partial Y^2}{\partial X^2}\Big|_{U^2=U^{2*}} = \frac{\frac{\partial U^2}{\partial X^2}}{\frac{\partial U^2}{\partial Y^2}} = \frac{p^x}{p^y}$$

The price system <u>decentralizes</u> the efficient allocation decision.