# Suggested Answers, Quiz 2**Form I**

Eight points possible. Median 6, average 5.5.

This is Form 1. Use this form if the **second** number in your student ID number is 1, 4, 7. Otherwise use another form.

For each of the statements below, circle (on the answer sheet) **TRUE** if the statement is always true, circle **FALSE** otherwise. $(P)$ refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and $(D)$ is the dual of $(P)$. Assume $(D)$ is feasible. $(P')$ refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq \alpha b, x \geq 0$$

where $1 > \alpha > 0$ and $(D')$ refers to the problem:

$$\min \alpha b \cdot y \text{ subject to } yA \geq c, y \geq 0$$

1. If $x^*$ solves $(P')$ and $y^*$ solves $(D')$, then $c \cdot x^* = y^* A x^*$. True. Part of Duality Theorem.

2. $(D')$ is the dual of $(P')$.

   True.

3. $(P')$ is feasible.

   False. We know $(D)$ is feasible and so $(D')$ is too feasible. But $(D')$ could be unbounded.

4. $(P')$ cannot be unbounded.

   True. Since we know $(D')$ is feasible.

5. If $(D')$ has a solution, then $(P)$ has a solution.

   True. $(D')$ is the same problem as $(D)$ (objective function multiplied by a positive constant). So if $(D')$ has a solution, then $(D)$ has a solution and so $(P)$ has a solution by duality theorem.

6. If $(P)$ is not feasible, then $(P')$ is not feasible.

   True. If $x$ satisfies constraints of $(P')$, then $x/\alpha$ satisfies constraints of $(P)$.

7. $(D')$ is feasible.

   True. $(D)$ and $(D')$ have the same feasible set.

8. If both $(P)$ and $(P')$ have solutions, then the value of $(P)$ is less than or equal to the value of $(P')$.

   False. The value of $(P')$ is smaller since $0 < \alpha < 1$.

# Form 2

This is Form 2. Use this form if the **second** number in your student ID number is 2, 5, 8, or 0. Otherwise use another form.

For each of the statements below, circle (on the answer sheet) **TRUE** if the statement is always true, circle **FALSE** otherwise. $(P)$ refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and $(D)$ is the dual of $(P)$. Assume $b \geq 0$. $(P')$ refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq \alpha b, x \geq 0$$

where $1 > \alpha$ and $(D')$ refers to the problem:

$$\min \alpha b \cdot y \text{ subject to } yA \geq c, y \geq 0$$

1. If $x^*$ solves $(P')$ and $y^*$ solves $(D')$, then $c \cdot x^* = y^* A x^*$.

   True. Part of Duality Theorem.

2. $(D')$ is the dual of $(P')$.

   True.

3. If $(D)$ is feasible, then $(P)$ has a solution.

   True. Duality Theorem says $(P)$ has a solution or is not feasible, but since $b \geq 0$, $(P)$ is feasible.

4. $(P')$ is feasible.

   False. If $\alpha < 0$ $(P')$ may be infeasible.

5. $(D')$ cannot be unbounded.

   False. This is possible if $\alpha < 0$.

6. If $(D')$ is not feasible, then $(P)$ has no solution.

   True. If $(P)$ had a solution, then so would $(D)$. So $(D')$ would be feasible (because it has the same feasible set as $(D)$).

7. If $(P)$ has a solution, then $(D')$ is feasible.

   True. Duality Theorem implies that $(D)$ is feasible and $(D)$ and $(D')$ have the same feasible set. This question is logically equivalent to the previous one.

8. If both $(P)$ and $(P')$ have solutions, then the value of $(P)$ is greater than or equal to the value of $(P')$.

   True. The value of $(P)$ must be nonnegative (since $x = 0$ is feasible). Either $\alpha \geq 0$, in which case If Value $(P') = \alpha$ Value $(P)$ (since $(D)$ and $(D')$ are equivalent problems) or $\alpha < 0$ and the solution to $(D')$ is $y = 0$ (since $y \geq 0$ is a constraint, it is not possible to do better).

# Form 3

This is Form 3. Use this form if the **second** number in your student ID number is 3, 6, 9, or if you have no student ID. Otherwise use another form.

For each of the statements below, circle (on the answer sheet) **TRUE** if the statement is always true, circle **FALSE** otherwise. $(P)$ refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and $(D)$ is the dual of $(P)$. Assume that $(P)$ is feasible.

$(P')$ refers to the problem:

$$\max \alpha c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

where $1 > \alpha > 0$ and $(D')$ refers to the problem:

$$\min b \cdot y \text{ subject to } yA \geq \alpha c, y \geq 0$$

1. If $x^*$ solves $(P)$ and $y^*$ solves $(D)$, then $c \cdot x^* = y^* A x^*$.

2. $(D')$ is the dual of $(P')$.

   True.

3. If $(D)$ is feasible, then $(D')$ is has a solution.

   True. $(P)$ and $(P')$ are equivalent problems (objective function multiplied by a positive constant). If $(D)$ is feasible, then $(P)$ must have a solution (since it is feasible). Hence $(P')$ must have a solution and so $(D')$ has a solution.

4. If $(D)$ is feasible, then $(D')$ is feasible.

   True. If $y$ satisfies the constraints in $(D)$, then $\alpha y$ satisfies the constraints in $(D')$. (Or: special case of previous question.)

5. If $(P)$ has a solution, then $(D')$ is feasible.

   True. $(P)$ has solution implies $(D)$ has a solution implies $(D)$ feasible. Implies $(D')$ feasible by the previous question.

6. If $(D')$ is not feasible, then $(P)$ has no solution.

   True. $(D')$ not feasible implies $(D)$ not feasible.

7. If $(P')$ has a solution, then $(D)$ is feasible.

   True. If $(P')$ has a solution, then so does equivalent problem $(P)$.

8. If both $(P)$ and $(P')$ have solutions, then the value of $(P)$ is greater than or equal to the value of $(P')$.

   True. The value of $(P') = \alpha$ value of $(P)$