Econ 172A, Fall 2014: Quiz I

IMPORTANT

Answers (with explanations) inserted below. Eight points possible (one point per answer). Median 5. Average 5.1. It looks like the second problem caused problems because several objective functions were correct. I apologize for any confusion, but the point is important: If you want to maximize a given linear function, then you could equally well maximize a positive multiple of the objective function or the objective function plus a constant.

1. The quiz has 3 forms. You should answer the questions from only one form.
   - If your student identification number ends in 1, 5, 7 answer the questions from Form 1.
   - If your student identification number ends in 2, 4, 9, 0 answer the questions from Form 2.
   - If your student identification number ends in 3, 6, 8, or if you have no student identification number, answer the questions from Form 3.
2. You may not use calculators, books, or notes during this quiz.
3. If you do not know how to interpret a question, then ask me.
4. Please remain in your seat until the exam is over.
5. You will not receive credit unless you put your answers in the spaces below.
6. I will collect the quizzes at 9:50.

RECORD ANSWERS

- NAME:
- STUDENT IDENTIFICATION NUMBER:
- I read the instructions and I am answering the questions corresponding to the appropriate form, which is FORM:

Question 1. Circle the correct choice or choices:	a.	b.	c.	d.
Question 2. Circle the correct choice or choices:	a.	b.	c.	d.

Form I

This is Form 1. Use this form if your student ID ends in 1, 5, 7. Otherwise use another form.

1. Consider the Linear Programming Problem:

$$\begin{array}{lcccc} \max & x_0 & & & \\ \text{subject to} & x_1 + 2x_2 & \leq & 4 \\ & x_1 - x_2 & \leq & 2 \\ & x & \geq & 0 \end{array}$$

.

Indicate on the front page which of the choices below are true. I recommend that you solve the problem by graphing the feasible set, but you need not show your work. (More than one statement may be true.)

(a) The feasible set has exactly four corners.
Yes. Graph it. The corners are $(0,2)$, $(2,0)$, $(8/3,2/3)$, and $(0,0)$

(b) It is possible to find a function $x_0 = c_1x_1 + c_2x_2$ (for some constants c_1 and c_2) such that the problem has no solution.
No. The feasible set is nonempty and bounded.

(c) No matter what x_0 is, adding the constraint $x_1 \leq 3$ will not change the solution to the problem.
Yes. The additional constraint is automatically satisfied (if the original constraints are satisfied).

(d) No matter when x_0 is, at least one of the points $(0,2)$, $(2,0)$, $(8/3,2/3)$, or $(2/3,8/3)$ is solution to the problem.
No. $(0,0)$ may be the unique solution (e.g. $\max -x_1 - x_2$).

2. Sinterklaas sells Christmas trees in December. He uses linear programming to describe his problem. Trees come in three sizes: large, medium, and small. The large trees cost \$35 and sell for \$ 60 each; the medium trees cost \$ 30 and sell for \$ 50 each; the small ones cost \$ 15 and sell for \$ 25 each. Sinterklaas must order at least twenty trees of each type. He can spent no more than \$ 3000 on investors of trees. His space limitations are such that he cannot exceed 60 units of the large and medium trees combined. He wants to obtain a gross revenue of at least \$ 5000 from selling trees. He wants to maximize his profits subject to these constraints. He formulates the problem as a linear programming problem, where the variables are x_L, x_M, and x_S, the number of large, medium, and small trees that he buys. His objective function is x_0. He writes his constraints as:

(a) $35x_L + 30x_M + 15x_S \leq 3000$.

(b) $x_L + x_M \leq 60$.

(c) $60x_L + 50x_M + 25x_S \geq 5000$.

(d) $x_i \geq 20$ (for $i = L, M, S$)

Indicate your answer on the first page.

Sinterklaas will maximize his profits if he solves the linear programming problem above using

(a) $x_0 = 60x_L + 50x_M + 25x_S$.
No. (This is revenue.)

(b) $x_0 = 25x_L + 20x_M + 10x_S$.
Yes. (This is the objective function.)

(c) $x_0 = 25(x_L - 20) + 20(x_M - 20) + 10(x_S - 20)$.
Yes. (This is the "true" objective function minus a constant.)

(d) $x_0 = 5x_L + 4x_M + 2x_S$
Yes. (This is a positive multiple of the "true" objective function.)

Form 2

This is Form 2. Use this form if your student ID ends in 2, 4, 9, 0. Otherwise use another form.

1. Consider the Linear Programming Problem:

$$\begin{array}{lrcrcl} \text{max} & x_0 & & & & \\ \text{subject to} & x_1 & + & 2x_2 & \leq & 4 \\ & x_1 & - & x_2 & \leq & 2 \\ & & & x & \geq & 0 \end{array}$$

.

Indicate on the front page which of the choices below are true. I recommend that you solve the problem by graphing the feasible set, but you need not show your work. (More than one statement may be true.)

(a) The feasible set has exactly four corners.

Yes. Graph it. The corners are $(0, 2)$, $(2, 0)$, $(8/3, 2/3)$, and $(0, 0)$

(b) It is possible to find a function $x_0 = c_1x_1 + c_2x_2$ (for some constants c_1 and c_2) such that $(2/3, 4/3)$ solves the problem.

Yes. The point is feasible (satisfies all constraints). (The only objective function that works requires $c_1 = c_2 = 0$.)

(c) No matter what x_0 is, adding the constraint $x_2 \leq 1$ will not change the solution to the problem.

No. The constraint shrinks the feasible set. (Consider $\max x_2$: the solution changes.)

(d) No matter when x_0 is, at least one of the points $(0, 2)$, $(2, 0)$, $(8/3, 2/3)$, or $(0, 0)$ is a solution to the problem.

Yes. These are the four corners.

2. Sinterklaas sells Christmas trees in December. He uses linear programming to describe his problem. Trees come in three sizes: large, medium, and small. The large trees cost \$50 and sell for \$ 110 each; the medium trees cost \$ 30 and sell for \$ 80 each; the small ones cost \$ 15 and sell for \$ 40 each. Sinterklaas must order at least fifteen trees of each type. He can spend no more than \$ 3000 on investors of trees. His space limitations are such that he cannot exceed 60 units of the large and medium trees combined. He wants to obtain a gross revenue of at least \$ 8000 from selling trees. He wants to maximize his profits subject to these constraints. He formulates the problem as a linear programming problem, where the variables are x_L, x_M, and x_S, the number of large, medium, and small trees that he buys. His objective function is x_0. He writes his constraints as:

(a) $50x_L + 30x_M + 15x_S \leq 3000$.

(b) $x_L + x_M \leq 60$.

(c) $110x_L + 80x_M + 40x_S \geq 8000$.

(d) $x_i \geq 15$ (for $i = L, M, S$)

Indicate your answer on the first page.

Sinterklaas will maximize his profits if he solves the linear programming problem above using

(a) $x_0 = 120x_L + 100x_M + 50x_S$.
Yes. (This is a positive multiple of the "true" objective function.)

(b) $x_0 = 60x_L + 50x_M + 25x_S$.
Yes. (This is the "true" objective function.)

(c) $x_0 = 25x_L + 20x_M + 10x_S$.
No. (This is revenue.)

(d) $x_0 = 60(x_L - 15) + 50(x_M - 15) + 25(x_S - 15)$.
Yes. (This is the "true" objective function minus a constant.)

Form 3

This is Form 3. Use this form if your student ID ends in 3, 6, 8, or if you have no student identification number.

Otherwise use another form.

1. Consider the Linear Programming Problem:

$$\begin{array}{rrcrcl} \max & x_0 & & & & \\ \text{subject to} & x_1 & + & 2x_2 & \leq & 4 \\ & x_1 & - & x_2 & \geq & 2 \\ & & & x & \geq & 0 \end{array}$$

.

Indicate on the front page which of the choices below are true. I recommend that you solve the problem by graphing the feasible set, but you need not show your work. (More than one statement may be true.)

   (a) The feasible set has exactly four corners.

   No. Graph it. The corners are $(4, 0)$, $(2, 0)$, and $(8/3, 2/3)$.

   (b) It is possible to find a function $x_0 = c_1x_1 + c_2x_2$ (for some constants c_1 and c_2) such that the problem has no solution.

   No. The feasible set is nonempty and bounded.

   (c) No matter what x_0 is, adding the constraint $x_1 \leq 3$ will not change the solution to the problem.

   No. The constraint shrinks the feasible set. (Consider $\max x_1$: the solution changes.)

   (d) No matter when x_0 is, at least one of the points $(4, 0)$, $(2, 0)$, $(8/3, 2/3)$, or $(2/3, 8/3)$ is solution to the problem.

   Yes. These include the three corners.

2. Sinterklaas sells Christmas trees in December. He uses linear programming to describe his problem. Trees come in three sizes: large, medium, and small. The large trees cost \$35 and sell for \$ 60 each; the medium trees cost \$ 30 and sell for \$ 50 each; the small ones cost \$ 15 and sell for \$ 25 each. Sinterklaas must order at least twenty trees of each type. He can spenh no more than \$ 3000 on investors of trees. His space limitations are such that he cannot exceed 60 units of the large and medium trees combined. He wants to obtain a gross revenue of at least \$ 5000 from selling trees. He wants to maximize his profits subject to these constraints. He formulates the problem as a linear programming problem, where the variables are x_L, x_M, and x_S, the number of large, medium, and small trees that he buys. His objective function is x_0. He writes his constraints as:

   (a) $35x_L + 30x_M + 15x_S \leq 3000$.

(b) $x_L + x_M \leq 60$.

(c) $60x_L + 50x_M + 25x_S \geq 5000$.

(d) $x_i \geq 20$ (for $i = L, M, S$)

Indicate your answer on the first page.

Sinterklaas will maximize his profits if he solves the linear programming problem above using

(a) $x_0 = 5x_L + 4x_M + 2x_S$.
Yes. (This is a positive multiple of the "true" objective function.)

(b) $x_0 = 60x_L + 50x_M + 25x_S$.
No. (This is revenue.)

(c) $x_0 = 25x_L + 20x_M + 10x_S$.
Yes. (This is the objective function.)

(d) $x_0 = 25(x_L - 20) + 20(x_M - 20) + 10(x_S - 20)$.
Yes. (This is the "true" objective function minus a constant.)