

Class Exercise 3

These questions come from 2010 Quiz 2, Form 1 with one modification: I added the $y \geq 0$ constraint in (D) .

Warning: It is easy for me to make slight modifications to the questions that lead to changes in the answer.

For each of the statements below, circle (on the answer sheet) **TRUE** if the statement is always true, circle **FALSE** otherwise. (P) refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0,$$

(D) refers to the problem:

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0,$$

and (P') refers to the problem:

$$\max c \cdot x \text{ subject to } Ax \leq b', x \geq 0$$

where $b' \geq b$.

Ways to approach the problem:

1. Two Tools: The Duality Theorem and Connection between Feasible Sets/Values of Different Problems.
2. Compare feasible sets of (P) and (P') (or (D) and (D')) to see if there are logical relationships (this allows you to use the second tool).
3. In particular, in this question (P') has a bigger feasible set than (P) , so if (P) is feasible, then (P') is feasible.

1. If (P) is feasible, then (P') is feasible.
2. If (D) has a solution, then (P') has a solution.
3. If (D) is not feasible, then (P') has no solution.
4. (D) is the dual of (P') .
5. If (P) is feasible, then (D) is feasible.
6. If both (P) and (P') have solutions, then the value of (P') is greater than or equal to the value of (P) .
7. If x^* solves (P) and y^* solves (D) , then $c \cdot x^* = y^* A x^*$.
8. If (D) is feasible, then (P') is feasible.

Preliminary observations:

A The feasible set of (P) is contained in the feasible set of (P') .

B The feasible set of (D) is the same as the feasible set of (D') .

[A] is true because $b' \geq b$. [B] is true because the only difference between (D) and (D') is the objective function.

Now some answers. I don't write "by the Duality Theorem" every time I use it. Sometimes I say "by DT."

1. True (from [A]).
2. True: (D) has solution means (D) feasible, so (D') is feasible (by [B]). Also (D) has solution means (P) has solution, so (P) is feasible so (P') is feasible by [A]. Since both (P') and (D') are feasible, both have solution.
3. True: (D) not feasible means (D') not feasible by [B]. Conclusion by DT.
4. False. (D) and (D') have different objective functions. (Warning: This would be true if $b' = \alpha b$, $\alpha > 0$. In that case (D) and (D') have equivalent objective functions.
5. False. (P) could be unbounded. (A one-variable example: $c = 1$, $A = b = 0$.)
6. True. (P') has a larger feasible set so the solution of (P) is feasible for (P') .
7. True. This is a consequence of DT. Discussed in class on 10-23.
8. False. (D) can be unbounded. (Like Part 5.)