Econ 172A, Fall 2010: Midterm A

Instructions

- The examination has 4 questions. Answer them all.
- To receive full credit, you must give brief justifications for your answers in Questions 1, 3 and 4. In Question 1 this means that you must state the reasons for all the inferences you draw for complementary slackness. In Question 3 you must explain why your explain does what you claim it does. Question 4 must explain how you arrived at your answers. Explains need not be long, but they should explain why you have answered correctly. No justification is needed for Question 2.
- You may not use books, notes, calculators or other electronic devices.
- Grading: There are 140 possible points. Point values are listed in the table below.
- If you do not know how to interpret a question, then ask me.

	Score	Possible
I		40
II		50
III		20
IV		30
Exam Total		140

1. Consider the linear programming problem:

   Find x_1, x_2 and x_3 to solve **P**:

$$
\begin{array}{lrcrcrcr}
\text{max} & 2x_1 & + & x_2 & + & 4x_3 & & \\
\text{subject to} & x_1 & + & 2x_2 & + & x_3 & \leq & 190 \\
 & 2x_1 & - & x_2 & + & x_3 & \leq & 65 \\
 & 2x_1 & + & x_2 & + & 5x_3 & \leq & 145 \\
 & & & x & \geq & 0 & &
\end{array}
$$

   You must provide justifications for your answers to the questions below. In particular, say what you need to do to check for feasibility and explain the basis for your inferences in part (c).

   (a) Write the dual of the problem **P**.

   (b) Verify that $(x_1, x_2, x_3) = (52.5, 40, 0)$ is feasible for **P**.

   (c) Assuming that $(52.5, 40, 0)$ is a solution to **P**, use Complementary Slackness to determine a candidate solution to the dual.

   (d) Is $(52.5, 40, 0)$ a solution to **P**? Explain.

   (e) Do you have enough information to find the solution to the dual of **P**? If so, find it.

2. I solved a linear programming problem written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0.$$

Attached find the Excel Answer and Sensitivity report. (I deleted some irrelevant information.) In these reports, I replaced several values with letters ((a) through (o)). Using the information in the table, replace as many of these letters with the correct information. You need not justify these answers (simply write the answers in the appropriate spaces, next to the letter). If you do not have enough information to figure out one or more of the values, write "NOT ENOUGH INFORMATION" next to the letter.

In addition to completing the tables, please answer the following questions. Once again, if you do not have sufficient information to answer a question, write "not enough information."

(a) How many variables are in the original problem (primal)?

(b) How many variables are in the dual?

(c) What is the objective function of the primal?

(d) What is the objective function of the dual?

(e) What is the value of the primal?

(f) What is the value of the dual?

(g) What is the solution to the primal?

(h) What is the solution to the dual?

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
E8	Large	60	(a)	5	1	(b)
F8	Medium	15	(c)	(d)	(e)	1.33333
G8	Small	0	(f)	4	4	(g)

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H16	Labor LHS	90	0	220	(h)	(i)
H17	Machine Time LHS	480	1	480	(j)	(k)
H18	Raw Material 1 LHS	100	0	3000	(l)	(m)
H19	Raw Material 2 LHS	-60	1	-60	(n)	(o)

3. Give an example of a linear programming problem that has a unique solution, but whose dual has more than one solution. (You can do this using a problem with one variable and one constraint (in addition to the non-negativity constraint).

4. Consider the linear programming problem: Find x_1 and x_2 to solve $\mathbf{P}$:

$$\begin{array}{lrcrcl} \max & Ax_1 & + & Bx_2 & & \\ \text{subject to} & x_1 & + & 2x_2 & \leq & 4 \\ & 2x_1 & - & x_2 & \leq & 4 \\ & -x_1 & - & x_2 & \leq & 1 \\ & & & x & \geq & 0 \end{array}$$

(a) Graph the feasible set of the linear programming problem. Clearly label the graph.

(b) Graphically solve the linear programming problem assuming:

i. $A = 1$ and $B = 1$

ii. $A = 1$ and $B = 4$.

iii. $A = 1$ and $B = -3$.

In each case state value of the problem, give the solution, and explain whether the solution is unique.

(c) When $A = 1$ give the smallest value of B with the property that $(0, 2)$ is the solution to the problem.

(d) Can $(.6, .6)$ be a solution to the linear programming problem? If so, then prove it by supplying an objective function for with $(.6, .6)$ is a solution. If not, explain why not.