Econ 172A, Fall 2004: Final Examination Suggested Answers

Comments: These are the answers to Form I. I explain the variations at the beginning of each answer.

There were 250 points possible on the exam. Scores ranged from 88 to 249 with a median of 189. Most students did extremely well on the first question. Scores on 2, 4, and 5 were fine. Problem 3 caused the most difficulty. On the third question, there were large deductions for not describing a plan that could solve the problem; on problem 4 it was important to answer each part consistently; the first and last parts of problem 5 caused problems. Students lost points for incomplete explanations.

1. Form II: I switched payoffs for Alex and Ryan and added 5 to all of Adam's benefits. So the answer is the same except that the benefit is higher by 5 and Alex and Ryan's assignments are reversed.

   First multiply all by negative one to change into a minimization:

	Bass	Drum	Guitar	Keyboard	Roadie
Adam	-10	-12	-6	-8	-5
Alex	-17	-13	-10	-16	-2
Chase	-15	-5	-18	-11	-6
Isaac	-14	-6	-16	-12	-4
Ryan	-14	-6	-16	-12	-10

   Second, add constants to each row to make the array non-negative. It is fine to add 18 (or more) to every element. To save time, I add the smallest number in each row to every element in the row. That is, 12 to the first row, 17 to the second, 18 to the third, and 16 to the last two rows. This guarantees both that all entries are non-negative and there is a zero in each row:

	Bass	Drum	Guitar	Keyboard	Roadie
Adam	2	0	6	4	7
Alex	0	4	7	1	15
Chase	3	13	0	7	12
Isaac	2	10	0	4	12
Ryan	2	10	0	4	6

   Third, subtract constants from each column so that there is a zero in every column (while maintaining non-negativity):

	Bass	Drum	Guitar	Keyboard	Roadie
Adam	2	0	6	3	1
Alex	0	4	7	0	9
Chase	3	13	0	6	6
Isaac	2	10	0	3	6
Ryan	2	10	0	3	0

Fourth, note that there is no zero-cost assignment available and "efficiently" cross out all of the zeros. In the array above, I crossed out the third column and rows 1, 2, and 5. Now (again using the array above) subtract the minimum uncrossed number (2) from everything and add 2 back into every crossed row and column to preserve non negativity:

	Bass	Drum	Guitar	Keyboard	Roadie
Adam	2	0	8	3	1
Alex	0	4	9	0	9
Chase	1	11	0	4	4
Isaac	0	8	0	1	4
Ryan	2	10	2	3	0

Now we have a zero-cost assignment: Adam-Drum; Alex-Keyboard; Chase-Guitar; Isaac-Bass; Ryan-Roadie. The benefit of this assignment (using numbers from the original table) is: $12 + 16 + 18 + 14 + 10 = 70$.

In the second part of the question, Chase must be assigned to drum. We can delete "Chase-Drum" from any of the tables above and continue with the algorithm. I started with the final array:

	Bass	Guitar	Keyboard	Roadie
Adam	2	8	3	1
Alex	0	9	0	9
Isaac	0	0	1	4
Ryan	2	2	3	0

The first row is this array is strictly positive, so subtract one from each element:

	Bass	Guitar	Keyboard	Roadie
Adam	1	7	2	0
Alex	0	9	0	9
Isaac	0	0	1	4
Ryan	2	2	3	0

There is no zero-cost assignment, so efficiently cross out all of the zeros (above); subtract the minimum uncrossed number (2) from everything; add 2 to each crossed row and column; and arrive at:

	Bass	Guitar	Keyboard	Roadie
Adam	0	6	1	0
Alex	0	9	0	10
Isaac	0	0	1	5
Ryan	1	1	2	0

This gives a zero-cost assignment: Adam-Bass; Alex-Keyboard; Isaac-Guitar; and Ryan-Roadie (of course, Chase is on the drums). The benefit of this assignment is: $10 + 16 + 5 + 16 + 10 = 57$. (Perhaps surprisingly, the benefit of the new assignment drops by more than the loss associated with shifting Chase from guitar to drums. The reason for that is that when Chase is on drums, the other boys must move to instruments that they do not play as well.)

I doubled the investment amount and the three costs. I changed the mix of regular to $3 : 7 : 1$. This makes only cosmetic changes to the answer.

2. Let x_j be the tons of mixture j produced, where $j = 1$ indicates regular; $j = 2$ for extra; and $j = 3$ for super. Notice that from x you can determine the amounts of the various ingredients you use, but not vice versa. (That is, if I reported that the company used so many tons of potash, nitrates, and phosphates, you would in general not have enough information to figure out what its final output was.) Also let a_{ij} be the tons of ingredient i used in product j. The first index, i, represents nitrates, phosphates, and potash for $i = 1, 2, 3$, respectively. The second index, j, indicates regular, extra, and super, respectively as before.

   There are simple expressions for the x_j in terms of the a_{ij}, namely, $\sum_{i=1}^{3} a_{ij} = x_j$. Also, we can write down profit. Revenue is simply

$$750x_1 + 800x_2 + 900x_3.$$

   While cost is

$$800 \sum_{j=1}^{3} a_{1j} + 400 \sum_{j=1}^{3} a_{2j} + 1000 \sum_{j=1}^{3} a_{3j}.$$

   Profit is the difference of these two quantities.

   We have the objective function. What are the constraints? The capacity constraint permits the firm to produce no more than 40 tons overall. It follows that

$$x_1 + x_2 + x_3 \leq 40.$$

The firm cannot spend more than 25000; consequently cost is less than or equal to 25000. Finally, the composition constraints need to be met. For the regular mixture we know that the ingredients appear in a 3:6:1 ratio. This means that $\frac{a_{11}}{a_{21}} = \frac{3}{6}$ and $\frac{a_{11}}{a_{31}} = \frac{3}{1}$. These equations can be written: $6a_{11} - 3a_{21} = 0$ and $a_{11} - 3a_{31} = 0.$. Observe that the composition requirement determines the quantity of two of the ingredients knowing the quantity of the other one. That is, if you fix a_{1j}, then you can deduce a_{2j} and a_{3j}. Noting similar constraints for the other products, we can summarize the formulation. We need to find the a_{ij} and the x_j to maximize profit (revenue minus cost above) subject to:

(a) $x_1 + x_2 + x_3 \leq 40$. (capacity)

(b) $800 \sum_{j=1}^{3} a_{1j} + 400 \sum_{j=1}^{3} a_{2j} + 1000 \sum_{j=1}^{3} a_{3j} \leq 25000$ (cost constraint)

(c) $6a_{11} - 3a_{21} = 0$.

(d) $a_{21} - 6a_{31} = 0$.

(e) $4a_{12} - 4a_{22} = 0$.

(f) $a_{22} - 4a_{32} = 0$.

(g) $4a_{13} - 6a_{23} = 0$.

(h) $3a_{23} - 4a_{33} = 0$.

(i) $a_{ij}, x_j \geq 0$

Without too much trouble, you can formulate this problem using only three variables: say the amount of nitrates used in each product. The mixing constraints would then determine the amount of other ingredients used in each product. Knowing these things, you can figure out the total weight of each product. Doing so yields the same problem. In my opinion, the formulation above is easier to understand (and easier to interpret). The lesson is that you can invent as many variables as you find useful. Extra variables probably will lead to extra constraints. On the other hand, naming all of the economically relevant terms may make the problem and its solution easier to interpret.

For the second part of the problem, the firm can earn ten percent return on the difference between 25000 and the cost of the ingredients. The constraints of the new problem don't change. Added to the objective function is the term: $.1(25000 - (800 \sum_{j=1}^{3} a_{1j} + 400 \sum_{j=1}^{3} a_{2j} + 1000 \sum_{j=1}^{3} a_{3j}))$. I changed the objective function to: $3x_1 - 4x_2 - 6x_3 + Ax_4$ in Form II. This modifies the CS argument by changing the inequalities to $y_1 - y_2 = 3$ and $-2y_1 + 3y_2 = A$. Solving yields: $y_1 = A + 9$ and $y_2 = A + 6$. So you need $A \geq -6$ to maintain dual feasibility.

3. The answer is $A \geq -\frac{5}{3}$. A way to arrive at the solution is to write down the complementary slackness implications, which are that the first and last dual constraints must bind. Solving these constraints yields $(y_1, y_2) =$

$(5{+}3A, 5{+}2A)$. Provided that $y \geq 0$, the second and third dual constraints are satisfied. Hence the conclusion holds when $5 + 3A \geq 0$.

4. (a) I increased all payoffs by 1 in Form II.

   Write down a payoff matrix for this game. (Label the strategies and explain what they represent.)

	1 PM	2 PM	3 PM
No	0	0	0
1 PM	-1	-1	1
2 PM	1	-1	-1
3 PM	1	1	-1

   The payoffs are payoffs to Jack. Rows represent Jack's strategies. Columns represent Jill's strategies. The strategies for Jack are when (if ever) to smoke. The strategies for Jill are when to check. Payoffs depend on whether Jill can detect Jack's smoking.

   (b) Does Jack have any dominated strategies? 2 PM is dominated by 3 PM (3 PM is strictly better unless Jill looks at 1 PM.)

   (c) Does Jill have any dominated strategies? Yes, check at 1 PM is dominated by checking at 2 PM.

   Note: in the last two parts it is possible to answer "no" if you take the definition of dominates to be "there is another strategy that is strictly better no matter what opponent does." This answer would receive credit provided that the definition was clearly stated (and you were consistent in your choice of definition across the two parts).

   (d) Find the pure-strategy security levels of both players. Jack's is 0 and Jill's is a payoff of 1 (to Row).

   (e) Does the game have an equilibrium in pure strategies? No, since the pure strategy security levels are not equal.

   (f) Now assume Jill's checking strategy is constrained in the following way: the probability that she checks at 1 PM must be equal to the probability that she checks at 2 PM. Write down a payoff matrix for this game. (To do this, assume that Jill has two pure strategies: "check at 3 PM" and "check at 1 PM with probability $\frac{1}{2}$ and check at 2 PM with probability $\frac{1}{2}$).

	Before 3	At 3
No	0	0
1 PM	-1	1
2 PM	0	-1
3 PM	1	-1

   (g) Find the equilibrium strategies and the value of the game you wrote in part (f).

   If Jill plays each of her strategies with probability one half and Jack does not smoke, then both players guarantee payoff zero.

(h) What does your answer to part (g) tell you about the equilibrium strategies and value of the original game? (Answer as completely as possible).

In the smaller game, Jill can guarantee a payoff of zero. She can't do worse than that in the big game. On the other hand, even in the original game, Jack can guarantee 0. So the value of the original game is zero. An equilibrium strategy is for Jack not to smoke while Jill checks at 1 PM with probability $\frac{1}{4}$; 2 PM with probability $\frac{1}{4}$; and 3 PM with probability $\frac{1}{2}$.

Form II is identical.

5. (a) This says that the amount of A produced is at least as large as the amount of A sold plus the amount of A used as input to production of B.

(b) Produce 20 units of A, use them as inputs for the production of B, produce and sell 10 units of B; earn 70.

(c) It does not change because 8 is less than the allowable increase of 9.5

(d) This is outside the allowable range, so the solution changes. It is still feasible to follow the original plan, which would lead to profit of \$550. This is greater than producing only A, so I think that production moves to C.

(e) The labor constraint binds and the shadow price is 17.5, so each additional labor hour is worth \$17.50, which is the most that the firm would pay for it.

(f) This lowers the objective function coefficient of T_A by one (to -1). This is in the allowable range. The solution stays the same and profits go down by 20 to 680.

(g) The value of the ingredients is $70 + 3(17.5) = 122.5$ since 70 is the shadow price on the drug B constraint and 17.5 is the shadow price of labor. Hence it is worthwhile producing the new drug if it can sell for more than \$122.50.

(h) This gives the value of an extra unit of drug C. That is, it allows the amount of Drug C sold to be different than the amount of Drug C produced. Notice that the shadow price is 100, which is exactly what you get for selling a unit of drug C. (The shadow price of drug A is higher than the price of drug A because it is used as an input in the production of something more profitable.)